ficiencies are apparent (thermal, phase change, etc.) and it is hoped that in lieu of perpetual constancy, the conclusions will stimulate and justify additional work in the same field by other investigators.

Acknowledgment

The contributions of F. Mannuzza in obtaining the experimental data and of W. C. Osborne in the writing of this paper are gratefully acknowledged.

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Discussion

H. E. Brandmaier.[5] Prior to his entry into the Navy the writer was very interested in the analysis of flow through close clearance pressure breakdowns, particularly in the work of the authors on that subject. They are to be complimented for the mathematical elegance of their presentation of the effect of eccentricity in the turbulent region.

In the way of criticism the writer feels that the authors' analytical treatment of the effect of rotation of one of the boundaries of the annulus is not altogether satisfactory. The conclusion that the addition of a rotational velocity component to the axial component has no effect in laminar flow can be deduced directly from a consideration of the Navier-Stokes' equations. Using the authors' assumptions, these equations reduce to two independent differential equations for the rotational and axial velocities. This result is independent of the geometrical transformation used and, if cylindrical co-ordinates are used, of the assumption that the flow closely approximates that between flat parallel plates. In addition, it has been the writer's experience that the addition of a rotational velocity component to a laminar flow in the breakdown usually changes the flow to transitional or turbulent flow. This restricts the applicability of the authors' Equation [17] to those cases where the flow is turbulent before and after the addition of the rotational component.

The writer would appreciate a more detailed evaluation of the effects of inlet and exit losses and methods of distributing the flow to the inlet (single and multiple holes feeding directly to the inlet or to an annular groove in one of the boundaries preceding the inlet, etc.) on the applicability of the results to cases where these factors are not the same as in the authors' experiments. In the case of centrifugal-pump wearing rings these effects may be equally as important as surface-friction effects.

Referring to the authors' Figs. 9 and 10, very little change in f is indicated for a fivefold relative roughness variation. In addition, the change appears to be in the wrong direction in Fig. 10 if a comparison is made with pipe-friction data.

The writer cannot agree with the authors that a 140-microinch finish with a 0.006-in. diametral clearance constitutes a hydraulically smooth surface.

In conclusion, the writer expresses the hope that the authors will extend their excellent work to include flow through serrated breakdowns which, in the turbulent region, are usually more effective than smooth-walled breakdowns.

O. E. Teichmann.[6] The authors are to be commended for a fine piece of experimental investigation that has yielded additional interesting data on flow through concentric and eccentric annular clearances. The idea of summarizing the results in the form of nomograms is certainly a lucky one and will be appreciated by the design engineer who needs a quick answer to the questions of leakage flow through annular gaps.

The presented experimental data are of special interest to the writer who took part in a similar investigation—although only on stationary arrangements without rotation of the shaft—which was carried out at Armour Research Foundation under an Air Force contract and which is planned for publication in the near future. Since this investigation centered around leakage flow in sleeve valves for control applications, radial clearances from 0.0003 to 0.002 in. were studied and it can be stated that, under laminar flow regime, the friction factor of 96/Re could always be verified experimentally when the entrance losses were not included in the measurement and true concentricity was established. For the turbulent region, the Blasius formula gave good agreement with the experimental values. In general, the experimental data recorded in the earlier literature were mostly carried out with considerable care and, occasionally, with more painstaking care than we are ready to apply today.

There is, of course, always the question whether we are looking for applied or fundamental data. The authors have chosen to investigate the applied case, limiting themselves to length/diameter ratios, clearances, roughness, and entrance conditions as practically encountered. This increases the value of the obtained data for the design engineer, but detracts from their value as basic information.

With regard to the mathematical analysis of the flow through an annular gap with rotational motion of the boundaries, it appears to the writer that the authors have made certain assumptions which would deserve closer scrutiny.

To clarify the picture, let us restate the various cases considered in the paper. We have a combination of two geometries (concentric and eccentric), two conditions at the boundaries (stationary and rotating), and, let us say, two flow regimes (laminar and turbulent), as shown in Table 1.

There are no reservations for any of the cases (marked with X) except for the eccentric cases (marked with ?). For the station-

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TABLE 1

		Laminar	Turbulent
Stationary	Concentric	X	X
	Eccentric	?	Becker developed equation but did not integrate it Schneckenberg integrated equation graphically Tao and Donovan: mathematical integration
Rotating	Concentric	X	X
	Eccentric	?	?

ary-eccentric-turbulent case, the authors have integrated mathematically the equations which had been previously set up by Becker (1907) and graphically integrated by Schneckenberg (1931). It is gratifying to have this mathematical solution recorded in the hydraulic literature.

For the concentric case, the combination of axial flow and rotational drag represents the actual flow phenomena quite adequately (helical flow path instead of straight axial flow).

For the eccentric case, however, the situation is more involved. Even with the boundaries stationary, the flow may be turbulent in the wide portions of the gap and change to laminar in the narrow portions. The friction coefficient, therefore, may not be consistent around the periphery. The error in axial leakage flow, however, may be negligible because only a small percentage of the total flow passes through the narrow regions of the gap.

If rotation is introduced in addition to the eccentric position and a generally turbulent flow regime, the picture gets considerably complicated. Assuming the flow moves along helical streamlines, the width of the flow passage, the velocity distribution, and the mean flow velocity in the gap, as well as the friction factor, will change along the path and the flow regime may change accordingly. The statement that "the problem with rotation can be treated as stationary by making an adjustment of velocity and flow configuration" is somewhat nebulous, and if the authors mean "an adjustment as used in the concentric case with rotation," the statement would be incorrect. It is, furthermore, not clear to the writer whether the authors wish to imply that the velocity distribution shown according to Pai for the concentric rotational gap also can be applied for the eccentric rotational case, which would definitely lead to basic contradictions (see velocity distribution in journal bearings).

In conclusion, it appears to the writer that the authors were more ambitious than necessary in adding to the very significant and valuable experimental investigation, an attempt of an analytical treatment of the eccentric case with rotation that is oversimplified to the extent where it cannot be expected to offer more than a very rough check of the experimentally determined data.

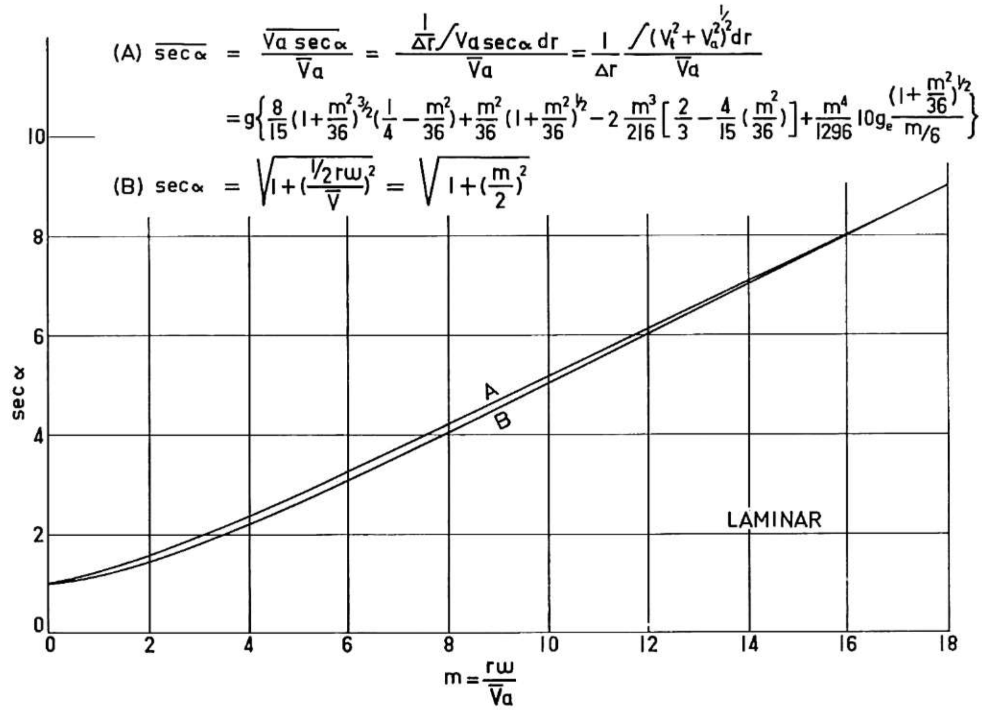

FIG. 14

AUTHORS' CLOSURE

The authors thank Messrs. Brandmaier and Teichmann for their comments and compliments. To answer their questions and discussion the following explanation is offered.

It is true that the statement that the addition of a rotational-velocity component to the axial component has no effect in laminar flow can be deduced from a consideration of the Navier-Stokes equations. It happens that in the case of laminar flow the pressure drop is linearly proportional to the mean velocity, or, we may say, "the linearity rule" is applicable in the laminar-flow regime. But it is not applicable to the turbulent-flow regime. The approach used by the authors was based on the change of the flow configuration due to the addition of the rotational velocity. This method of determining the mean velocity has been checked with the mathematical average obtained from the local resultant velocities and also affirmed by experiment. With the integration of the local resultant velocities of both tangential and axial velocity components, the mean velocity of laminar-flow regime across the annular space has been calculated as a function of the ratio of the velocity components in both tangential and axial direction, which is plotted in Fig. 14 as curve *A*. This was compared with the approach adopted in the paper, which is plotted as curve *B* in Fig. 14. The agreement between the curves appears to justify the authors' simplified approach. Unfortunately, a similar computation for the turbulent zone was not possible due to the lack of available information.

Equation [17] is valid, of course, only so long as Equation [15] is true. This requires that the n-value with and without rotation be identical if the form of Equation [17] is employed. This will not hold for the case where the flow condition is changed from one to the other but if necessary, Equation [15] is available for this case.

Regarding the inclusion of entrance and exit losses in the investigation, it has been stated in the paper that this is significant but necessary since the primary object of this investigation was the determination of the fluid flow across an annular-flow channel due to the total pressure difference available. During the course of the investigation the authors felt that for the cases without the inclusion of the entrance and exit losses the results for the rather simple case can be either directly or indirectly deduced from the known results in literature, e.g., in the stationary and concentric cases they can be obtained from the equation $f = 96/\text{Re}$ in laminar-flow regime and from the Blasius formula of friction in the turbulent regime. While on the other hand there is no well-established information available for the case with the inclusion of the entrance and exit losses. This led to the primary objective of the present investigation. Furthermore, the mathematical derivations have no restriction whatsoever to the length-diameter, entrance, roughness, and some other conditions, if the proper constants and coefficients are inserted.

Mr. Teichmann's point on the flow in the eccentric annular channel with rotation is well taken. The authors agree that the flow analysis for a rotating eccentric channel is very complicated and the dangers of overgeneralization are always present. However, for an idealized case where the effect of rotation is not to change the over-all n-value, the results as presented in the paper are believed to be consistent and applicable to many actual design problems. The adoption of the velocity profile for the eccentric annular space with rotation given in the paper is practically feasible. Strictly speaking, the velocity in the tangential direction should be the superposition of the velocity due to the rotation and that due to the pressure gradient along the circumferential direction created by the variation of the local gap. But the term of this pressure gradient is small compared to the terms of drag and the pressure gradient in the axial direction, and it is negligible from the consideration of the order of magnitude. This notion has been employed by Ocvirk[7] in the lubrication study of journal bearings and its result was affirmed by experiment (for detail the work by Ocvirk and the references therein may be referred to).

It was commented that 140 microinches would not constitute an hydraulically smooth surface. This was the authors' semantics. What the authors want is to define a smooth surface as distinguished from a serrated surface.

The change of slope in Fig. 10 has been discussed during the presentation of the paper. As we all know that, in the turbulent pipe-friction data, the slope for the perfectly smooth surface should be at an angle of arctan 0.25 in a logarithmic f-Re diagram and horizontal for the surface of extremely high roughness, i.e., the n-value of Equation [10d] in the paper should be 0.25 for the former and zero for the latter. For all other roughnesses n should fall between these two extreme values. In the present problem with 140 microinches, it is expected that the n-value will fall between zero and 0.25.

[7] "Short-Bearing Approximation for Full Journal Bearings," by F. W. Ocvirk, NACA Technical Note 2808, October, 1952.