frequency would result in a decrease in actual amplitude for the same forced frequency and harmonic torque values. On the other hand, with m/p originally greater than 1, then the actual amplitude is increased as a result of the change. This can be verified by use of Equation [7] of the paper and the effect of such changes is shown in Fig. 4 of the paper. Mr. Pyles' remarks concerning the conflicting nature of the requirements of frequency and amplitude are undoubtedly based on a desire to keep the running range as free from criticals as possible. In synchronous-speed machinery, this is not essential since operation at one speed requires passage through the criticals only at starting and stopping, and, provided the stimulating harmonics are not too large, it is possible to accelerate or decelerate through the objectionable speed ranges without noticeable ill effects. The advantage of operating in ranges of m/p greater than 1 is obvious from Fig. 4 of the paper; the matter in any case is usually one of compromise and, with some latitude in choice of mass and elasticity, a satisfactory arrangement can usually be made.

Mr. Magdeburger's description of the effect of a flywheel on the nodal position is applicable only when the engine, without a flywheel, is not coupled to an external mass. Operating in this condition has no practical significance in so far as generator units are concerned for an external mass is always present. Study of a system consisting of an engine and a generator driven by means of a shaft coupled to the crankshaft flange will show that the possibility of the node falling outside the engine exists; and for normal proportions the probability of occurrence is high (see Fig. 3 of Mr. Pyles' discussion). The interposition of a flywheel at the crankshaft flange reduces this probability and the node usually falls within the engine, if by the latter is meant the engine crankshaft. Mr. Magdeburger further reviews the procedure for determining minimum shaft diameters. This point is so axiomatic that in oral discussion the author felt it unnecessary to refer to it. A review of generator shaft sizes used in practice and the minimums predicated by strength considerations will show a wide range of flexibilities to be feasible.

The author agrees with Mr. Schweizer as to the necessity of providing sufficient mass for proper speed control under load changes. Under such conditions, provided a modern fuel system is employed, the behavior of the unit is a function of governor response alone; hence the lower mass limit for satisfaction in this respect is usually specified by the governor manufacturer. It is assumed that in mass elastic calculations the basic mass limits previously enumerated are respected.

The question of the relative merits of the various means by which satisfactory regulation may be achieved is, to the author at least, an open question. There are undoubtedly many cases where the conventional arrangement may prove most satisfactory and economical. The use of an outboard flywheel does present difficulties, some of which are of a mechanical nature. However, the alternative of placing the flywheel inboard and as close to the generator as practical so as to make both virtually one mass appears quite promising and cases may arise where either of these two constructions may prove better than the conventional.

Determination of Rate of Discharge in Jerk-Pump Fuel-Injection Systems[1]

C. R. Alden.[2] When the writer started to work with fuel-injection phenomena, he found a similarity to the transient phenomena existing in the electric circuit, wherein velocity of flow in the pipe has its direct counterpart in inductance, and compressibility or elasticity having its counterpart in capacitance. Experience with the hydraulic phenomena was a help toward understanding the electrical, and the groundwork of electrical training is quite a help in understanding what takes place in the hydraulic circuit.

Hans Fischer.[3] Everybody who works on high-speed Diesel engines is confronted with problems of hydraulics in the injection system, and we can never get uniform hydraulics over a large speed range of the engine.

The writer recently ran a series of tests with a certain nozzle and a certain nozzle spring. The test was run at engine speeds ranging from 600 to 2700 rpm. The combustion conditions were very satisfactory, except at 1600 rpm. At this speed, the load had to be reduced about 20 per cent in order to obtain a clean exhaust. We then tried another nozzle of similar type, but with a needle about 30 per cent heavier. With this nozzle, the erratic condition at 1600 rpm disappeared and the required load could be run without smoky exhaust. The writer would like to ask the author about vibration problems in hydraulics connected with the spring-loaded needle. Also, what is the effect of a constant nozzle orifice as used in the hole-type nozzle when compared with the pintle-type nozzle with its variable orifice?

Carl Behn.[4] It would be interesting to know the comparison between the author's calculations as to duration and injection pressures and the actual measured results. That would probably tie in with Mr. Fischer's questions. Has the author made comparisons of specific installations; that is, applications of various pumps, delivery valves, delivery lines, and nozzles with various pressures for definite results of pressure rises and injection durations? How do these compare with the calculated pressure rise and duration?

Dana W. Lee.[5] The writer recently assisted in building a special injection valve which will measure the pressure at the nozzle. When we have secured some data with this nozzle there will be a very close check with either the graphical method of analysis or the mathematical method which has been adapted to this work The stem of this injection valve is of the usual differential-area spring-loaded type, but a $^1/_8$-in. hole was drilled along its axis for the entire length and a steel plunger placed in it. There is a close lap fit between the valve stem and the plunger so that the stem is free to lift during the injection period while the plunger remains stationary. One end of the plunger is exposed to the high-pressure fuel in the space between the stem seat and the nozzle orifice, and the other end rests against the piezoelectric crystal pickup of a cathode-ray oscillograph.

Thus, we are able to measure the instantaneous pressure variations at the entrance to the nozzle bore, and from these pressures the discharge rates may be easily computed by the usual flow formula. Variations in the discharge rate too slight to be detected by any other method are fully shown on the oscillograph screen. Friction between the stem and the plunger will introduce some error, but this should not be serious. Our greatest difficulty will be to get a true calibration of the oscillograph. If we can do that we will have a very sensitive and accurate

[1] Published as paper OGP-60-2, by K. J. DeJuhasz, in the February, 1938, issue of the A.S.M.E. Transactions.

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[3] Lanova Corporation, New York, N. Y.

[4] National Supply Company, Springfield, Ohio.

[5] National Advisory Committee for Aeronautics, Langley Field, Va.

record of the injection pressure at the discharge orifice. We have made a preliminary check with satisfactory results.

O. L. Riegels.[6] The following comments do not refer to the author's method of presentation, but rather to the objectives which must be reached through this analysis. From observance of various phenomena given considerable thought, and often confirmed by experience, the writer has found the aspect of the fluctuations is more complicated than generally supposed. The writer finds it logical that two major and different wave series are originated in a pump-injection system.

The first will have the character of a pressure wave, originated by a pump stroke and in a time much longer than $2L/a$, or pipeline period, and therefore of less violence. This wave is started some time before the injection valve opens.

The other wave series is originated at the nozzle tip when the injection valve opens and the fuel column under pressure tries to regain its original density.

In modern nozzles, with light moving parts, the opening time is only a fraction of $2L/a$ and this pressure-drop wave must therefore inherently be proportionally more violent. There probably will be no separation occurring in the column, but there must be some kind of contraction in the column itself.

That the pressure-drop wave is the most powerful can be noted from the fact that the injection lag in good designs usually correspond to L/a.

Although the two different wave series have equal speed of propagation and equal harmonics, the timing and intensity are different, and they are originated at opposite ends. To balance these against each other, as the writer has seen proposed, will therefore not be possible.

Oscillographs of the fluctuations usually show unsymmetrical forms and wavefronts that only can be accounted for as a result of more than one wave series.

Another condition to be taken into account is the fact that the velocity of efflux starts at zero and accelerates as an exponential curve against time, and in the short injection time never reaches steady flow conditions.

L. H. Donnell.[7] The author neglects the expansion of the pipe in comparison with the contraction of the fluid. There are probably other approximations of even more importance which are unavoidable in such a complex problem. However, it is of some interest that this approximation can be largely eliminated by use of a corrected fluid modulus. For thin-walled pipes[8]

$$k' = \frac{k}{1 + (kd/Et)} \qquad [1]$$

where k is the volume modulus of the fluid, k' is the volume modulus approximately corrected for the effect of pipe flexibility, E is the tensile modulus of the pipe material, and t and d the wall thickness and average diameter of the pipe, respectively.

For thick-walled pipes, the tangential and radial stresses at the inner wall due to an internal pressure p are

$$\sigma_t = \frac{\beta^2 + 1}{\beta^2 - 1}\, p,\ \sigma_r = -\,p \qquad [2]$$

where β is the ratio of the outer diameter to the inner diameter.

[6] Consulting Engineer in charge of Diesel Engine Development, Yoder Company, Cleveland, Ohio. Mem. A.S.M.E.

[7] Engineer in charge of Stress Analysis, Goodyear-Zeppelin Corporation, Akron, Ohio. Mem. A.S.M.E.

[8] "Longitudinal Wave Transmission and Impact," by L. H. Donnell, Trans. A.S.M.E., vol. 52, 1930, paper APM-52-14, pp. 153–167.

The effective unit-volume change due to the contraction of the fluid and expansion of the pipe cross section (the effect of longitudinal strain in the pipe is somewhat complex and is not important for thick-walled pipe) is then

$$\frac{p}{k'} = \frac{p}{k} + \frac{2}{E}(\sigma_t - \mu\sigma_r) \qquad [3]$$

from which, taking Poisson's ratio $\mu = 0.3$

$$k' = \frac{k}{1 + \dfrac{k}{E}\,\dfrac{2.6\ \beta^2 + 1.4}{\beta^2 - 1}} \qquad [4]$$

It is interesting to note that even if β were infinite, a correction would be required of about 2.5 per cent for steel pipe and 5.5 per cent for brass or bronze pipe with fuel oil. For pipes designed with a reasonable factor of safety, the correction may of course be considerably higher.

If such parts as reservoirs and pump cylinders were made of the same material as the pipe and designed with similar factors of safety, the foregoing value of k' could be used with good approximation for the complete system.

Author's Closure

Mr. Alden is quite right in pointing out the close analogy between the transient phenomena in liquid columns and in electric circuits. Disturbances in linear elastic systems, be they mechanical (spring surges), hydraulic (water-hammer in conduits and injection phenomena), pneumatic (surges in intake and exhaust pipes), and electric (transient phenomena in electric circuits) form one great family and obey the same basic laws.

It is quite conceivable that the application of such refined analytical tools as Steinmetz' method of complex variables, and Heaviside's operational calculus—which are used to great advantage in electrical problems—would prove equally fruitful of results in the field of transients in hydraulic or mechanical systems. Nevertheless, it appears to the author that their abstractness sets them at a disadvantage in the eyes of a mechanical engineer, in comparison with the graphical method used in this paper which gives at a glance a complete picture of the whole progress of the phenomenon. Conversely, the graphical method could probably be applied to electrical systems—if it has not been done already.

Referring to Mr. Fischer's discussion the influence on the spray of varying spring characteristics and needle mass can be taken into consideration in the graphical analysis, as it is shown in some simple examples in one of the author's other papers.[9] While diagnosis is not yet therapy, nevertheless it is a first step to it, and it is to be hoped that the graphical analysis will prove itself useful in designing nozzles with specified performance characteristics.

Referring to Mr. Behn's question, our experiments were in agreement, in the main, with the prediction of the graphical analysis. Detail differences can be accounted for by the simplifying assumptions which are necessary in all kinds of theories, and from which the present analysis is no exception. Such simplifications were the neglect of enlarged volumes in the pipe, the inertia of the needle valve, viscosity of the liquid, and leakage. These could be taken into consideration by refinements in the analysis at the cost of added labor. No theory for a physical phenomenon can be expected to express fully the reality, yet a clear comprehension of the theory is an indispensable pre-

[9] "Graphical Analysis of Transient Phenomena in Linear Flow," by K. J. DeJuhasz, *Journal of The Franklin Institute*, vol. 223, April, 1937, pp. 463–493; May, 1937, pp. 643–654; and June, 1937, pp. 751–788.

requisite to any rational experimentation or development. One of the objectives of our present program of research, which is yet in its early stage, is to determine how far it is legitimate to introduce simplifications.

It is gratifying to learn from Mr. Lee's remarks that the N.A.C.A. continues the investigation of injection to which subject they made such numerous and important contributions, and especially that an experimental checking of the analytical method is to be undertaken. All of us who are engaged in this field of research look forward with great interest to the outcome of such investigations.

The author is in agreement with Mr. Donnell's correction. The bulk modulus of the liquid is

$$k = -V\,(dp/dV) \qquad [5]$$

while for the purposes of our analysis, strictly speaking, the longitudinal compression modulus of the liquid column

$$k' = -l\,(dp/dl) \qquad [6]$$

should be considered. The two quantities are interrelated by the equation

$$V = l\,(d^2\,\pi/4) \qquad [7]$$

In a yielding pipe both l and d change with pressure according to

$$\frac{dV}{dp} = \frac{\pi}{4}\,2ld\,\frac{dd}{dp} + d^2\,\frac{dl}{dp} = -\frac{\pi}{4}\,\frac{ld^2}{k} \qquad [8]$$

Substituting

$$\frac{dd}{dp} = \frac{d^2}{2Et} \quad \text{and} \quad \frac{dl}{dp} = -\frac{l}{k'}$$

we obtain

$$k' = \frac{1}{(1/k) + (d/Et)} \qquad [9]$$

With this value, the acoustic velocity in the column will be $a' = \sqrt{(k'V)}$, and the proportionality factor of pressure change to velocity change will be

$$\frac{\Delta p}{\Delta v} = \frac{k'}{a'} = \tan\alpha \qquad [10]$$

with which the graphical construction has to be performed.

In the paper, for simplicity's sake, this distinction between k and k' was omitted in view of the practically nonyielding property of the thick-walled pipes used in fuel-injection systems; however, in case of thin-walled yielding pipes, the corrected k' and a' values should be used.

U. S. Navy Correlation of Laboratory Tests on Diesel Fuels With Service-Engine Operation[1]

F. L. Garton.[2] Looking at the author's fuel-performance curves on his various engines, we observe, for example, that in Fig. 10 for the Winton engine, he obtains severe knock with a computed combustion shock of about 1.75, whereas in Fig. 9 for the Buda engine anything below about 8 gave smooth combustion; that is to say, this figure for combustion shock by itself is not an indication of the value of a fuel unless you also consider the engine in which it is determined. It is important to distinguish carefully between the effect of the fuel on the engine and the effect of the engine on the fuel.

H. A. Everett.[3] Referring to the top curve of physical-chemical fuel-ignition indexes shown in Fig. 2 of the paper, the writer believes that the data could be better represented by a straight line rather than by the reverse curve used by the author. It would be interesting to know why the author chose to draw the reverse curve rather than a straight line.

C. W. Good.[4] The writer would appreciate knowing whether or not the author's curves, showing pressure variation in the cylinder, represent the actual pressure throughout the cylinder or merely pressure waves which impinge upon the indicator. The writer believes that when the so-called roughness of the curves, due to the delay in combustion, is obtained a mean of the pressures in the cylinder at any instant with this kind of combustion would be less than that obtained with the better fuels; that is, there is a definite energy loss with rough combustion which results in a lower effective pressure on the piston. Has the author any information on this point?

F. G. Shoemaker.[5] It is unfortunate that in conducting some of these fuel tests the pressure-indicator element was attached at the end of a tube of considerable length. For measurements where the rate of pressure rise is small, this arrangement introduces little error, but for high-speed engines, such as the Buda automotive type and the larger Winton two-cycle single-cylinder engine, the use of any tube length at all leaves the indicator card open to question as to the validity of any rate of pressure-rise measurements, such as are necessary in computing the combustion shock.

It is hoped that other papers now being prepared on the effect of indicator-tube length will show the magnitude of the probable errors in the measurements reported in the present paper,[1] and that the tests can be repeated with improved instrumentation which will separate the characteristics of the fuel from the characteristics of the engine.

Harte Cooke.[6] For a long time there has been the question as to just how the results obtained on a very small cylinder correlate with that which happens in a large cylinder. There is not only the difference in the action of the fuel in actual operation, but there is the difference in the testing. In a small cylinder, the cooling factor is very great in proportion to the volume; also, the nature of the combustion in regard to combustion delay and knock rating will vary between the large and small cylinders.

For example, in a cylinder of 18 in. diameter, the cooling factor is entirely different from that in a small cylinder, and no one knows just how the combustion process takes place in this larger cylinder. When the program reported in this paper[1] was initiated, the writer suggested that engine test cylinders be used which would give the difference between those various sizes. This was done but they only went to an 8-in. cylinder, and the tests with this cylinder apparently show there are differences in the test results with the 8-in. cylinder as against the very small one. Also, there are differences in the operating results between the larger and smaller engines.

[1] Published as paper OGP-60-3, by W. F. Joachim, in the February, 1938, issue of the A.S.M.E. Transactions.

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