gear caused by the turbine torque is 0.035 radian; the amplitude of vibration is also equal to 0.035 radian (at the gear), giving a total maximum deflection of 0.07 radian (1.65 in spring deflection), which corresponds to a maximum stress of about 32,000 lb. per sq. in. in each spring. This stress is the true maximum stress at the outer fiber on the inside of the coil. It is composed of static stress of 16,000 lb. per sq. in. and a variable stress which has an amplitude of 16,000 lb. per sq. in. These stresses are very conservative for the spring steel used.

The spring stops limit the maximum travel to 3 in., with a corresponding maximum stress of 53,500 lb. per sq. in.

Conclusion

It has been shown that it is possible to predict the suitability of any chosen gear flexibility if the drawings of the rotating parts for calculating the critical speeds and indicator cards (theoretical or actual) at two or three speeds are available.

The calculation made in designing the springs for the ship installation were in some error because the natural frequencies were calculated too high by some 20 per cent. This can be attributed to lack of experience in calculating such marine installations, and in nowise vitiates the value of the method to those practiced in the art of calculating such things.

Discussion

Hans Bauer.[7] For purposes of discussion this paper may be considered under two headings: first, the application of exhaust turbines in reciprocating-engined steamships, and second, the mathematical theory of oscillations in elastic rotating systems.

Concerning the first classification, the writer is able to offer observations based upon the successful operation of over 200 vessels equipped with exhaust turbines constructed according to the Bauer-Wach system and aggregating approximately 1,000,000 hp. In that system, two single-reduction gears are connected by a Vulcan hydraulic coupling, the latter being interposed between the high-speed gear shaft and the low-speed pinion shaft. The hydraulic coupling serves both to absorb shocks and vibration and as a clutch to disconnect the turbine and high-speed gear from the engine and low-speed gear when maneuvering, or at other times when it is desired to run on the engine alone.

Reversing is best performed when the exhaust turbine is disconnected, since it is practically impossible to synchronize the admission of steam to turbine and reciprocator as required for maneuvering, quite apart from the complicated design that it necessitates. Therefore, if the turbine is kept continually coupled to the engine, the gear teeth and the whole system are repeatedly subjected to reversal of stresses of unknown magnitudes. It may be claimed that this can be overcome by using the turbine alone when maneuvering. To do so, however, restricts the maneuvering power to that available from the turbine, and there are still torque variations due to dragging the engine. The much simpler method is to disconnect the turbine and use the engine for maneuvering.

That the hydraulic coupling provides complete protection of the turbine and gear system against torsional shocks or vibrations originating in the engine and propeller system may be seen from torsiographs which have been published from time to time. This result is considered ample justification for a slippage which, be it noted, constitutes about one-half of 1 per cent of the total power.

To introduce springs into an oscillating system is to invite trouble from vibration. By proper proportioning of one to the other the period may be controlled, but the tendency to oscillate remains and will be felt at some frequency. The authors admit that it is impossible to design a spring coupling to damp out torsional vibrations at all speeds, and they should not be satisfied if they have succeeded in doing this in or near the usual running range of the engine, since, as all marine engineers know, a marine engine may be called upon to run at any speed up to the maximum, according to weather conditions, load, etc., and if torsional vibration occur even for only a short period, it may be detrimental to the gearing. Conditions in an oscillating system like the above linked by a flexible and elastic member will be much more difficult to determine, and almost impossible to control, if the propeller is momentarily lifted out of the water in a seaway.

It is not quite clear what the authors mean by "propeller damping." The degree of propeller damping must necessarily be different under different draft and weather conditions. The propeller cannot prevent resonance as between the turbine system and the engine system. Springs interposed between the two systems obviate the extreme stresses due to short-period oscillations, but are not effective—in fact, they constitute a source of danger when the frequency of the impressed forces approaches the natural frequency of oscillation of the spring-coupled system.

A hydraulic coupling, on the other hand, not being elastic, does not transmit any torsional oscillations.

The authors are to be commended for having gathered and analyzed mathematical researches relating to torsional vibration and the design of spring couplings. To criticize a detail, however, it does not seem advisable to the writer to base designs on stresses as high as 32,000 lb. per sq. in. in spring steel, since the springs may at times be compressed to the maximum travel when, as stated by the authors, the stress would be raised to 52,500 lb. per sq. in. This may occur frequently, as, for example, with a racing propeller in rough weather, or, again, in maneuvering.

R. Eksergian.[8] The authors are to be congratulated on an excellent technical investigation of a complicated problem. An outstanding feature of the presentation is the clearness in picturing the phenomena with skilful approximations.

The normal elastic curves in Fig. 8 show very clearly the advantage of elasticity at the coupling in producing large amplitudes for the propeller damping, which, from Equation [26], shows corresponding reduction in the amplitude of vibration caused by a given disturbing moment.

The writer has been further impressed with the advantage of reducing the elasticities and inertias to a common shaft coordinate. In this way a much clearer picture of the relative inertias and elasticities of the system is obtained. The turbine rotor of only 2120 lb-in.-sec.^{2} polar moment of inertia when reduced to the propeller-shaft coordinate and thereby increased by the square of the products of the gear ratio becomes 3,900,000 lb-in.-sec.^{2} Thus the turbine becomes practically the pivot point of the system. Likewise the turbine spindle, which is fairly flexible in itself, becomes extremely stiff when multiplied by the square of the products of the gear ratio and reduced to the propeller shaft.

The authors' analysis for thus placing the flexibility in the main gear is very interesting. The ratios stated for geometrical spring systems appear obvious except the last one. A little consideration, however, will show that $\alpha_1 = (r_2/r_3)\alpha_2$ is correct if the amplitude at the propeller-shaft junction is to remain the same. We have the two cases shown by Fig. 13.

If the amplitudes α_D at the junction of the propeller shaft remain the same for the two cases, then, with the assumption of the negligible amplitude of the reduced turbine mass due to its enormous value, the authors show that on the basis of a geometrical-similarity comparison, the stresses and torques trans-

[7] 11 Broadway, New York, N. Y.

[8] Consultant, Eng. Dept., E. I. du Pont de Nemours & Co. Wilmington, Del. Mem. A.S.M.E.

mitted are considerably increased when the spring system is placed in the high-speed gear. On the other hand, referring to Fig. 8, the amplitudes at the propeller-shaft junction on the basis of unit amplitude at the engine decrease with increased stiffness of the spring system. But the decreased amplitude at the propeller, however, for the stiffer spring system may result because

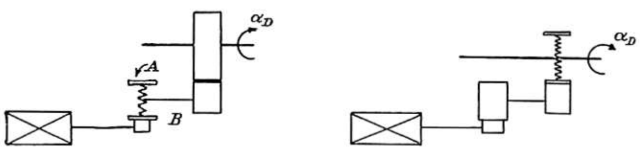

FIG. 13

CASE I

$$\alpha_D = \alpha_c$$
$$\alpha_c r_2 = \alpha_B r_3$$
$$\alpha_B - \alpha_4 = \alpha_1$$
$$\text{If } \alpha_4 = 0,\ \alpha_B = \alpha_1$$
$$\therefore\ \alpha_D = \alpha_1 \frac{r_3}{r_2}$$

CASE II

$$\alpha_D - \alpha_c = \alpha_2$$
$$\alpha_c r_2 = \alpha_B r_3$$
$$\alpha_B = \alpha_4 = 0$$
$$\text{If } \alpha_c = 0,\ \alpha_D = \alpha_2$$
$$\therefore\ \alpha_2 = \alpha_1 \frac{r_3}{r_2}$$

of the consequent decreased damping at resonance, in possibly larger actual amplitudes for the system under the exerting engine torque. Therefore α_D for the stiffer spring, as in Case 1, may be further augmented as compared with α_D in Case II, which would appear even more to favor the authors' conclusions.

Another point of view in addition to the authors' proof of a condition of inherent high stresses, etc. in the secondary gear is on the basis of the dynamics of the system by a comparison of the elastic constants of the springs for the two cases.

If T is the torque transmitted and ϕ the corresponding relative angular displacement between the spider and gear rim, then the elastic or spring constant is

$$K = \frac{T}{\phi} = \frac{nPR}{\dfrac{8NPD^3}{d^4GR}} = \frac{nR^2d^4G}{8nD^3}$$

Using the authors' geometrical-similarity assumption and noting that the reduced spring constant of the secondary gear is $K_1' = K_1 (r_2/r_3)^2$, we have the ratio of the elastic constants

$$\frac{K_1'}{K_2} = \frac{n_1R_1^2d_1^4G_1}{8N_1D_1^3}\left(\frac{r_2}{r_3}\right)^2 \bigg/ \frac{n_2R_2^2d_2^4G_2}{8N_2D_2^3} = \left(\frac{r_1}{r_2}\right)^3\left(\frac{r_2}{r_3}\right)^2$$

The greater the flexibility, the *smaller* the spring constant; therefore, if the ratio C_1'/C_2 is greater than unity, the greatest flexibility is obtained by placing the springs in the primary or low-speed gear. If, on the other hand, the factor $(r_2/r_1)^3 > (r_2/r_3)^2$, the greatest flexibility is obtained in the secondary gear. For this latter condition to be realized, assuming the gear ratio $r_2/r_3 = 5.8$, then $r_1/r_2 < 0.31$, which from constructional limitations alone it is impossible to obtain. With the practical value of $r_1/r_2 = 0.679$, the ratio of the spring constants is $K'_1/K_2 = 10.5$. This shows, when considering the flexibilities of the system, the considerable advantage in placing the springs in the low-speed gear.

The actual torques transmitted for the two cases are: $K_2\,\alpha_{D2}$ with spring in low-speed gear, and $K_1\,(r_2/r_3)\,\alpha_D$ with spring in high-speed gear. Hence the torques transmitted for the two cases are

$$\frac{T_1}{T_2} = \frac{K_1\left(\dfrac{r_2}{r_3}\right)\alpha_{D1}}{K_2\alpha_{D2}} = \frac{\left[K'_1\bigg/\left(\dfrac{r_2}{r_3}\right)^2\right]\dfrac{r_2}{r_3}\,\alpha_{D1}}{K_2\alpha_{D2}} = \left(\frac{r_1}{r_2}\right)^3\frac{r_2}{r_3}$$

which agrees with the author's conclusions, provided $\alpha_{D1} = \alpha_{D2}$.

In estimating the modes of oscillation corresponding to the natural frequencies of the system, the authors have shown that the problem is concerned with the second mode of oscillation. The reduced dynamical system shown in Fig. 6 has five degrees of freedom, and by using the Holzer method the two lower frequencies can be accurately estimated by trial solutions.

By further combinations of the inertias and elasticities we can reduce the system to one of two degrees of freedom, which would be in fair agreement with the first mode and approximately accurate for the second mode when compared with the system of five degrees of freedom. Such a resolution, however, may be of interest.

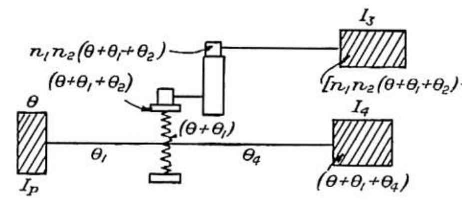

FIG. 14

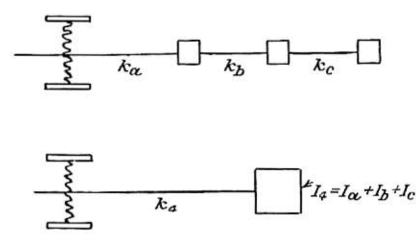

FIG. 15

The approximate two-degree system is shown in Fig. 14. The inertias of the gearing when reduced to the line shaft are small compared with the inertia of the turbine. Therefore they may be neglected or added to the turbine inertia I_3. The inertia of the propeller is I_p, and that of the engine, I_4.

To approximate the elastic constant of the engine shaft to the gearing, if k_a, k_b, and k_c are the elastic constants shown in Fig. 15 between the gearing and engine masses I_a, I_b, and I_c,

$$\frac{1}{k_4} = \frac{\dfrac{I_a}{k_a} + \left(\dfrac{1}{k_a} + \dfrac{1}{k_b}\right)I_b + \left(\dfrac{1}{k_a} + \dfrac{1}{k_b} + \dfrac{1}{k_c}\right)I_c}{I_a + I_b + I_c}$$

Now considering the system shown in Fig. 14, we have for the kinetic and potential energies,

$$T = {}^1\!/_2\, I_p\theta'^2 + {}^1\!/_2\, I_3\,(n_1n_2(\theta' + \theta_1' + \theta_2') + \phi_3')^2 + {}^1\!/_2\, I_4\,(\theta' + \theta_1' + \theta_4')^2$$

$$V = {}^1\!/_2\, k_1\theta_1^2 + {}^1\!/_2\, k_2\theta_2^2 + {}^1\!/_2\, k_3\phi_3^2 + {}^1\!/_2\, k_4\theta_4^2$$

where θ' and ϕ' are the first derivatives of θ and ϕ with respect to time.

The momentum corresponding to the coordinate θ is a constant, which we may take equal to zero, i.e.,

$$I_p\theta' + I_3\,(n_1n_2\,(\theta' + \theta_1' + \theta'_2) + \phi_3')n_1n_2 + I_4\,(\theta' + \theta'_1 + \theta'_4) = \theta$$

from which we can eliminate the coordinate θ in the expression of the kinetic energy. Or we may differentiate directly, so that,

$$(I_p + (n_1n_2)^2\,I_3 + I_4)\theta'' = -\,n_1n_2I_3(n_1n_2(\theta_1'' + \theta_2'') + \phi_3'') - I_4\,(\theta_1'' + \theta_4'') \ldots\ldots [33]$$

from which we can express the second derivative ϕ'' in terms of θ_1'', θ_2'', θ_4'', and θ_3''.

The equations of motion corresponding to the coordinates θ_2 and θ_3 are,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \theta'_2}\right) = n_1n_2I_3\,[n_1n_2(\theta'' + \theta_1'' + \theta_2'') + \phi_3''] = -k_2\theta_2 \quad [34]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \phi_3'}\right) = I_3\,[n_1n_2(\theta'' + \theta_1'' + \theta_2'') + \phi_3''] = -k_3\phi_3 \quad [35]$$

which, on combining, give

$$k_2\theta_2 = n_1n_2k_3\phi_3, \qquad \therefore\ \theta_2 = \frac{n_1n_2k_3}{k_2}\phi_3$$

that is, the torque of the elastic gear balances the reduced inertia torque of the turbine shaft. This provides additional elimination of the coordinates.

The two remaining equations are,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \theta_1'}\right) = nn_2I_3\,[n_1n_2(\theta'' + \theta_1'' + \theta_2'') + \phi_3''] + I_4\,(\theta'' + \theta_1'' + \theta_4'') = -k_1\theta_1 \quad [36]$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \theta'_4}\right) = I_4\,(\theta'' + \theta_1'' + \theta_4'') = -k_4\theta_4 \quad [37]$$

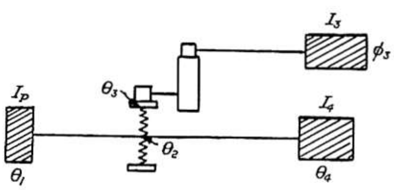

Fig. 16

Combining [34] and [37] with [36], we note that

$$k_1\theta_1 = k_2\theta_2 + k_4\theta_4$$

which we should anticipate immediately from the condition of equilibrium at the coupling. Therefore

$$\theta_1 = \frac{k_2}{k_1}\theta_2 + \frac{k_4}{k_1}\theta_4 = n_1n_2\frac{k_3}{k_1}\phi_3 + \frac{k_4}{k_1}\theta_4$$

On substituting the values of θ_1'' and θ_2'' in terms of ϕ_3'' and θ_4'' in [33], we have

$$\theta'' = -A\phi_3'' - B\theta_4''$$

where

$$A = \frac{n_1n_2\left[\left((n_1n_2)^2\left(\frac{k_3}{k_1} + \frac{k_3}{k_2}\right) + 1\right)I_3 + \frac{k_3}{k_1}I_4\right]}{I_p + (n_1n_2)^2 I_3 + I_4}$$

$$B = \frac{(n_1n_2)^2\frac{k_4}{k_1}I_3 + \left(1 + \frac{k_4}{k_1}\right)I_4}{I_p + (n_1n_2)^2 I_3 + I_4}$$

Substituting the values of θ'', θ_1'', and θ_2'' in terms of ϕ_3'' and θ_4'' in Equations [35] and [37], we obtain

$$\left[1 + (n_1n_2)^2\left(\frac{k_3}{k_2} + \frac{k_3}{k_1}\right) - n_1n_2A\right]I_3\phi_3'' + n_1n_2\left(\frac{k_4}{k_1} - B\right)I_3\theta_4'' + k_3\phi_3 = 0$$

$$\left(n_1n_2\frac{k_3}{k_1} - A\right)I_4\phi_3'' + \left(1 + \frac{k_4}{k_1} - B\right)I_4\theta_4'' + k_4\theta_4 = 0$$

The frequencies are obtained from the solution of the determinate

$$\begin{vmatrix} k_3 - \omega^2\left[1 + (n_1n_2)^2\left(\frac{k_3}{k_2} + \frac{k_3}{k_1}\right) - n_1n_2A\right]I_3, & -\omega^2n_1n_2\left(\frac{k_4}{k_1} - B\right)I_3, \\ -\omega^2\left(n_1n_2\frac{k_3}{k_1} - A\right)I_4, & k_4 - \omega^2\left(1 + \frac{k_4}{k_1} - B\right)I_4 \end{vmatrix} = 0$$

On substituting for the numerical values given by the authors for the actual installation, a good check is found with the frequency values obtained by them. A closer approximation would be obtained by including the inertias of the gearing with an additional degree of freedom.

The determinate, however, is of use for comparing different spring constants and in approximating the frequencies for similar installations.

In order to calculate the ratio of the amplitudes a different system of coordinates is used as shown in Fig. 16. The equations of motion, with these coordinates are

$$I_4\theta_4'' = k_4(\theta_4 - \theta_2)$$

$$I_3\phi_3'' = -k_3\,(\phi_3 - n_1n_2\theta_3)$$

$$I_p\,\theta_1'' = k_1\,(\theta_2 - \theta_1)$$

and

$$k_2\,(\theta_3 - \theta_2) + k_4\,(\theta_1 - \theta_2) - k_1\,(\theta_2 - \theta_1) = 0$$

$$-k_2(\theta_3 - \theta_2) + n_1n_2k_3\,(\phi_3 - n_1n_2\theta_3) = 0$$

If we assume the amplitude at the engine equal to unity, then

$$\theta_2 = \frac{k_4 - I_4\omega^2}{k_4}, \qquad \theta_1 = \frac{k_1\,(k_4 - I_4\omega^2)}{k_4\,(k_1 - I_p\omega^2)}$$

$$\theta_3 = \left(\frac{k_1 + k_2 + k_4}{k_2}\right)\theta_2 - \frac{k_1}{k_2}\theta_1 - \frac{k_4}{k_2}$$

$$\phi_3 = \left(\frac{k_2 + (n_1n_2)^2 k_3}{n_1n_2k_3}\right)\theta_3 - \frac{k_2}{n_1n_2k_3}\theta_2 = \frac{n_1n_2k_3\theta_3}{k_3 - \omega_2I_3}$$

The last equation could also be used for calculating the frequencies.

Heinrich Holzer.[9] To find out the best way of coupling an exhaust turbine of steady-torque characteristic to a reciprocating engine of variable-torque characteristic we may start by considering the fact that vibration in the turbine shafting can arise only from the coupling linkage. Therefore, if the point of the engine shafting at which the turbine branch is attached has no vibratory motion itself, the whole turbine branch will remain without vibrations.

The turbine branch should be attached to the main system at a point where a node of the vibration of the main system occurs. In general the vibration of the propeller system will consist of harmonic components of different periods and phases, and a coupling point without any vibratory motion will not be available. In this case we must of course be content to couple the two systems at one of the nodes of the main harmonic component. At the coupling point the amplitudes of both the main shaft and the turbine branch are not only the same, but also the harmonic coupling torques are equal but of different signs, representing

[9] Consultant on Vibration and Strength Problems, Schwabach, Germany.

the action—and reaction—torque. By means of these harmonic coupling torques both the main shaft and the turbine branch can be treated as free systems independent of each other. At the turbine branch the harmonic coupling torque is the only exciting torque determining completely the vibratory motion of the turbine-gearing system. Now it is well known that of all vibratory states of a given system, the natural or free vibrations of the system demand the smallest exciting torques down to zero, if there is no damping in the system. The effect of the reciprocating engine on the turbine branch, represented by the harmonic coupling torque, will therefore be eliminated when the turbine-gearing system is tuned to the frequency of the main component vibration of the engine-propeller system. When, for instance, the plant is ordinarily running between 85 and 89 r.p.m. and the main vibration is of third order, we may tune the turbine-gearing system to the free frequency of $3 \times 87 = 261$ cycles per min., if there is not critical speed in or near the working range; we may tune it to $3 \times 80 = 240$ cycles per min. if there is a heavy critical speed of the third order at 80 r.p.m. The flexibility of the springs coupling the low-speed gear to the propeller shaft is therefore of little or no account, though it is never wrong to make the springs as flexible as possible. When running at the speed to which the turbine-gearing system is tuned, there will be no vibration torque whatever in the gearing nor in the springs connecting the low-speed gear to the propeller shaft.

Of course, the best solution is to tune the turbine branch to the best running conditions and at the same time attach it to the propeller shaft at the point of least amplitude of vibration.

Frank M. Lewis.[10] The paper directs attention to the dynamic problems which arise when reciprocating machinery is used in conjunction with geared transmission. As the authors state, it is necessary that a positive torque be always maintained at the gear faces of such an installation. The irregular torque of the reciprocating machinery can start oscillations in the system which may cause the torque at the gear teeth to pass through negative loops. The pounding thus caused results in rapid deterioration of the gears. It is very difficult to insure this condition of positive torque without the introduction of a considerable amount of elasticity at an appropriate point in the system. In all the installations with which the writer is familiar, elasticity in some form was introduced—hydraulic, electrical, or in the form of mechanical springs. In the installation described springs have been used and the writer believes that the authors have effected a very satisfactory solution of the problem.

Propeller damping coefficients are deduced from the so-called propeller characteristics. The damping coefficient is essentially the slope of the torque–angular-velocity curve. It is this slope which the authors deduce in their Equations [10] to [19]. However, the writer is not quite in agreement with their results. The curve they give is based on an assumption that the ship's speed increases as the revolutions increase, while the slip increases only slightly. The condition of a vibrating propeller differs widely from this, however. Here the ship's speed remains constant while the slip is varying widely as the propeller vibrates. A torque–angular-velocity curve for this condition would have a much steeper slope than the one corresponding to Fig. 11, with a consequent higher damping coefficient. The writer deduced a method of calculating propeller damping coefficients based on these latter assumptions in 1925. While he has no data relative to the propeller dealt with in the paper, the method would probably give a damping coefficient 50 per cent to 75 per cent higher than the one calculated by the authors.

Any check of propeller damping coefficients by direct experiment is a very difficult matter. The best that can be done is to make torsiograph measurements on actual ships where all conditions and exciting forces are known. It is not an easy matter to make tests under such satisfactory conditions that reliable information will result.

In regard to the inertia effect of the water, our knowledge of propellers is also very inadequate, even more so than in regard to the damping effects. It is usual to make a purely arbitrary allowance of some 10 to 25 per cent of the propeller inertia, according to the taste of the calculator. The writer has made many torsional vibration investigations on ships, but has never been able to obtain a sufficiently comprehensive set of observations to permit an accurate calculation of either the inertia or damping effects of the water.

C. Richard Soderberg.[11] On page 8 there is an attempt to determine the damping properties of a ship propeller from the speed-horsepower curve. Now, a speed-horsepower curve such as the one shown in Fig. 11 is based on a steady state of motion, that is, the speed is assumed to be constant for each horsepower reading. This is so important that great care has to be taken in guarding against speed variations during the test. In view of this, it does not appear permissible to draw conclusions of the damping properties from this curve. For the damping phenomenon we are interested in the behavior of the propeller under cyclic variations of the speed; the speed-horsepower curve is entirely unrelated to this phenomenon.

Just what ought to be the correct relation for the damping resistance is difficult to say, but it is probable that it will depend upon the velocity of the oscillation $d\alpha/dt$ as well as on the mean speed of rotation N. The comparatively satisfactory agreement of tested and calculated amplitudes must be regarded as accidental.

O. G. Tietjens.[12] In this paper an attempt has been made to derive the damping constant for small torsional vibrations of a propeller shaft from a curve representing the indicated horsepower as a function of the revolutions per minute of the propeller. Leaving the question open for a moment as to whether the curve shown in Fig. 11 of the paper is the right one or not, it may be of interest to explain in general how the propeller supplies the damping, or in what manner the amount of energy that corresponds to the damping is being dissipated by the propeller.

If we consider the performance of a propeller-blade element having the peripheral speed u and the axial speed w, Fig. 17, we see that the relative speed of the blade element with respect to the water will be $v = \sqrt{(u^2 + w^2)}$. Each blade element has a certain angle of attack α with respect to the relative velocity v. The torsional vibration of the propeller shaft produces a periodic

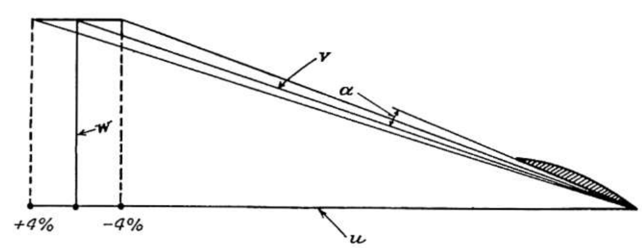

Fig. 17

(The variation of the peripheral speed u of a blade element caused by torsional vibration of the propeller increases and decreases periodically the angle of attack of the blade element.)

[10] Consulting Engineer and Professor, Webb Institute of Naval Architecture, New York, N. Y. Mem. A.S.M.E.

[11] Development Division Engineer, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa. Mem. A.S.M.E.

[12] Research Laboratories, Westinghouse Electric & Manufacturing Co., East Pittsburgh, Pa. Mem. A.S.M.E.

increase and decrease in the peripheral speed u of each blade element. The amplitude of this variation in u is about ± 4 per cent in the actual case dealt with in the paper. Therefore, since the axial velocity w remains constant during this periodic variation of the peripheral speed, we see from Fig. 17 that the torsional vibration of a propeller shaft produces a similar variation in the angle of attack of each blade element. These changes of the angle of attack are combined with a small variation of the relative velocity v as shown in Fig. 17. However, the variation of the angle of attack is the much more important factor.

On the other hand, an increase in the angle of attack of a blade element means an increase of the lift of this element, and a decrease in the angle of attack means a decrease of the lift. However, we know[13] that an increase in the lift generates behind the blade element a vortex which rotates in a counterclockwise direction if the blade element moves from the right to the left, and we know that a decrease of the angle of attack causes a vortex which rotates in a clockwise direction. Therefore we see that the torsional vibration of the propeller shaft gives rise to a periodic generation of vortices as shown in Fig. 18. The kinetic energy of these vortices produced per unit time corresponds to the work of damping done by the propeller.[14]

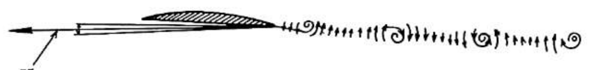

Fig. 18
(The kinetic energy of the vortex path produced per unit time corresponds to the work of damping done by the propeller.)

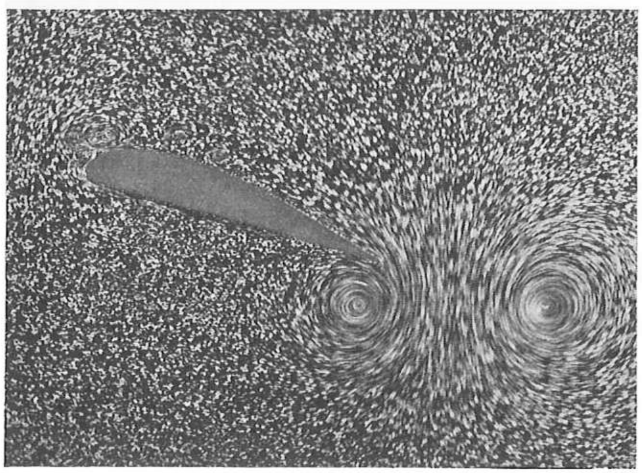

Fig. 19
(Showing two vortices which are generated behind a blade section by a variation in lift of this blade.)

Fig. 19 shows a picture of a wing section with two vortices generated by a variation in lift. The vortex at the right corresponds to an increase of the lift from zero to a certain amount; the vortex close to the trailing edge of the wing section corresponds to a decrease of the lift to zero. Due to the fact, however, that for the propeller-blade elements the variation in lift caused by the torsional vibration is much smaller than that corresponding to Fig. 19, the vortices for the propeller blades will also be much smaller than shown in this figure.

However, it is important to realize that a certain time is required to enable these vortices to reach the size that corresponds to the amount of variation in the angle of attack. If the frequency of the torsional vibration of the shaft, and therefore the frequency of the variation of the angle of attack, becomes too high, say, 100 cycles per second, the time for developing the vortices will be too short and very little or no energy will be dissipated. For very high frequencies, therefore, the propeller will supply no damping at all. In this case the tangent to the curve in Fig. 11 should be a horizontal line and $C = dT/d\omega = 0$. For a smaller frequency of variation the angle of attack C has a certain finite value. Therefore it is seen that the damping constant must be considered an unknown function of the frequency f, or

$$C = \frac{dT}{d\omega} = F(f)$$

However, even if we make the assumption that the frequency of the torsional vibration of the shaft is small enough to guarantee the generation of the vortices to their full size, it is further necessary to take into account a certain relation between the hydrodynamical forces and the frequency. This relation depends on the expression $\pi l/tv$, where l is the width of the blade, v the relative velocity of the blade element, and t the time of one cycle. The smaller the above expression, the less does the lift of a blade element, and therefore also the thrust as well as the torque, depend on the frequency. However, according to a theory developed by W. Birnbaum,[15] even for

$$\frac{\pi l}{tv} = 0.15$$

the amount of the variation in the lift, and therefore also in the torque, is about 20 per cent smaller than that based on the assumption of a steady flow, i.e., for $\lim 1/t = 0$. Assuming the data given in the paper, i.e., a blade width of about 2 ft., a frequency of 4 cycles per second, and a relative velocity of 4200 ft. per min. or 70 ft. per sec., we obtain approximately

$$\frac{\pi l}{tv} = 0.3$$

which makes it evident that the performance of a propeller with such a variation of its angle of attack will be different from that taking place under steady-flow conditions.

However, even if we have a sufficiently low frequency, or high velocity v, so that the above expression becomes small enough to justify the assumption of steady flow around the blades, the calculation of the damping constant should be based on a curve which gives the indicated horsepower as a function of the propeller revolutions for *constant* ship speed instead of the curve of Fig. 11 of the paper which gives the indicated horsepower as a function of the propeller r.p.m. where the ship speed varies with the r.p.m. As also mentioned by Professor Lewis, this has to be taken into account since the ship speed remains constant while the peripheral speed of each blade element varies due to the torsional vibration of the propeller shaft.

The curve for constant ship speed, however, is steeper and has a larger tangent than the curve of Fig. 11 at points of corresponding values of propeller revolutions. This can easily be seen from Fig. 17 if we realize that with increasing r.p.m. the increase in the angle of attack, and hence the increase in the torque, would be smaller if the speed of the ship, and therefore the axial velocity w increases with increasing propeller revolutions.

If we could make the assumption that the performance of the vibrating blade can be approximated by the performance of the blade in steady flow, we could then obtain the above-mentioned

[13] O. G. Tietjens, "Hydro- und Aeromechanik nach Vorlesungen von L. Prandtl," Vol. II, pp. 180–184, Berlin, 1931.

[14] It may be mentioned that these vortices have nothing to do with the so-called Kármán vortex path.

[15] W. Birnbaum, "Das ebene Problem des schlagenden Fluegels," *Zeit. f. Angew. Math. u. Mech.*, 1924, p. 277.

curve of indicated horsepower as a function of propeller revolutions for constant ship speed if the r.p.m. is suddenly increased or decreased and the indicated horsepower is then determined before the ship has had time to change its speed.

However, since we do not know in what manner the damping depends on the frequency, or more generally, on the term $\pi l/tv$, the derivation of the damping constant from a curve similar to Fig. 11 does not appear to be based on sound physical principles.

These remarks should not be considered as a criticism of this interesting paper as a whole, but as referring only to that part of it which deals with the theoretical derivation of the damping constant.

Authors' Closure

The primary purpose of the Vulcan coupling in the Bauer-Wach exhaust turbine is to permit the disengagement of the low-pressure turbine during maneuvering. Incidentally, it may minimize vibrational effects. All the published studies of this coupling concern themselves with its steady-state operation at constant rotational speed on both sides of the coupling. The Vulcan coupling must have distinct reactions to vibratory motion. There must be a loss of energy associated with oscillatory relative motion between the parts; and there is a torque between the driving and driven members that depends on their relative displacement. This elastic torque would be analogous to the synchronizing torque found in synchronous electrical machinery.

No quantitative values of these damping and elastic torques are known when the coupling is applied. The application may be effective against the vibration or it may not; that depends on the relationship of the vibratory properties of the coupling to the rest of the system.

As a damping device the Vulcan coupling is in its best position on the high-speed shaft. As an elastic device it might as well be torsionally rigid in this position. Without quantitative knowledge of its damping properties, reference must be made to experience. If it is a universally effective protective device against vibration, the gear teeth in all the 200 Bauer-Wach installations will only be worn or wire-edged on the driving side. If it is not, signs of pounding will appear on both sides of the teeth. The authors have been informed that such pounding and wear on both sides of the gear teeth is in evidence in some installations where the Vulcan coupling is used.

The spring gear described in the paper eliminated signs of pounding on both sides of the gear teeth.

It must be remembered that gear-tooth pounding and ultimate breakage are usually fatigue effects. Destructive effects require time and the repeated action of millions of blows. If these blows can be eliminated from the speed range where the ship runs 95 per cent of the time, the number of blows has been reduced to such an extent that other factors become more important in the ultimate life of the equipment. It is also understood that the Bauer-Wach system must be disconnected at lower speeds on account of pounding in the gear teeth.

Propeller damping is a very real thing. It is so effective that the one-noded vibration has practically no amplitude even at resonance. The purpose of a spring coupling is to keep amplitudes of stress harmless even in the event of resonance. Experience shows that it accomplishes this end.

The working spring stresses amount to 16,000 lb. per sq. in. static with an oscillating stress with 16,000 lb. per sq. in. amplitude; these stresses are well within the limit of safety of the material used as one year's continuous service amply proves. The maximum stress of 52,500 lb. per sq. in. occurs at low speeds and is not repeated often. It is also within the limits of safety for the material.

Dr. Eksergian's mathematical treatment reducing the complicated system to one of a few degrees of freedom is very interesting. It can be very useful in calculating the values of the natural frequencies after the numerical data on inertias and spring constants have been obtained.

Mr. Holzer's contention that the whole problem of oscillatory gear stresses could be eliminated by placing the gearing at a node in the elastic system is entirely correct. If the second reduction gear could have been placed at the node of the two-noded mode of motion, the gears would need no elasticity to protect them. Unfortunately insurance regulations and considerations other than torsional vibration determine the masses of the steam engine and propeller and the size of the main propeller shaft. The properties of most ship installations are such as to put the important node somewhere in the propeller-shaft alley far aft of the available engine-room space. These practical reasons are determining factors, especially when the exhaust turbine is installed in a ship originally designed to operate only with a multiple-expansion steam engine.

Dr. Holzer's statement that tuning the turbine and gear system would entirely eliminate any gear-torque reactions at those speeds where the harmonic frequencies were in resonance with the tuned frequency of the branch system, is correct. However, unless this tuned frequency is exactly equal to the 2-noded natural frequency of the system, it does not offer any great advantage since the maximum force in the gear teeth grows very rapidly as the 2-noded frequency recedes in either direction from the tuned frequency of the branch system. Practically, this means that the natural frequency of the branch system would have to be adjustable since it is almost impossible to calculate within limits of 5 to 10 per cent of the actual value of the 2-noded critical speed of one of these installations.

In the case of the *Susan V. Luckenbach* the 2-noded natural frequency is somewhere in the order of 4 cycles per second, which would require a flexibility between the turbine and gearing about 50 times as great as that of the present pinion shaft. Such a flexible coupling would certainly require more space than is used at present between the turbine and the gearing. The spring gear as actually used takes up no more space than an installation which has no flexibility in it. In the engine room of a ship, space is at a great premium.

With adjustable flexibility in the turbine coupling, Dr. Holzer's scheme is not only theoretically correct, but probably would offer actual practical advantages. Without adjustability and with complete dependability on design calculations, we feel that the spring gear used by us in the *Susan V. Luckenbach* probably gives a better practical answer.

The spring gear as used is not only a theoretical cure for a difficult problem, but it works.

Messrs. Lewis, Soderberg, and Tietjens have attacked the authors' method of calculating the propeller damping. The authors cannot defend the method used on physical grounds and can only add their desires to those of the discussors for a good theoretical and experimental method for determining the actual values for propeller damping. They cannot agree with Mr. Soderberg that the method is completely valueless, however.

First of all, the damping constant found by the authors' method agrees in form with those found by Frahm and Lewis:

$$\text{Lewis:}\quad C = \left(1 - S\right)\frac{\frac{\partial T}{\partial s}}{T}\;\frac{T}{\omega}$$

$$\text{Frahm:}\quad C = r\,\frac{T}{\omega}$$

$$\text{Ormondroyd and Kuchler:}\quad C = \left(n - 1\right)\frac{T}{\omega}$$

where T = propeller torque (steady rotation)
ω = angular velocity of propeller (steady rotation)
S = slip of propeller
r = Frahm's constant
n = exponent of N (r.p.m.) in expression for propeller torque (steady rotation).

The disagreement between these methods concerns only the value of the constant, which in the authors' method is 3. Lewis, in his paper cited on page 5, gives a method for calculating this constant from propeller-model tests and based on good physical reasoning. Using data published by Porter[16] based on the Lewis method, this constant should be about 3.7 for the propeller in question, although Porter's data refer to three-bladed propellers and the propeller in the authors' case has four blades. Frahm's constant as published by Porter in the same paper was 3.8.

General considerations indicate that the damping constant found by the authors' method will always be too small. This leads to the prediction of larger amplitudes than actually exist, which is on the safe side as far as design prediction is concerned.

The method proposed by Lewis is the best available, although Tietjens indicates that the problem is even more complicated than indicated by Lewis. What is really needed for a large class of ship-machinery vibration problems is an experimental study of propeller damping effects.

[16] F. P. Porter, "Practical Determination of Torsional Vibration in an Engine Installation," Trans. A.S.M.E., 1929, paper APM-51-52, p. 241.