TABLE 1 MODULI OF ELASTICITY OF SPECIMENS TESTED

Specimen	Material	Temp., deg. fahr.	Modulus
	MEASUREMENTS MADE USING AMSLER MACHINE		
111	0.68% C heat-treated steel wire, 1/8 in. × 1/2 in.	76	29,700,000 − 7.7 × S
112	Do.	76	30,100,000 − 6.8 × S
113	Do.	76	29,500,000 − 8.3 × S
530, 538	3 1/2% nickel steel 1/2 in. diam.	78	29,140,000 − 4.3 × S
476	Do.	73	28,930,000 − 10.9 × S
306	Spring-tempered phosphor-bronze, 1/8 in. × 1/2 in.	72	14,680,000 − 10.8 × S
	MEASUREMENTS MADE USING DEAD-WEIGHT LOADING[1]		
481	0.67% C heat-treated steel wire, 0.0281 in. diam.	62.6	30,040,000 − 6.6 × S
479	Do., 0.0465 in. diam.	69.8	30,160,000 − 6.5 × S
483	17 SRT aluminum-alloy wire, 0.065 in. diam.	72.5	10,280,000 − 10.9 × S
493	Spring-temper brass wire as drawn, 0.040 in. diam.	72	14,610,000 − 7.1 × S
	Do., after 0.22 per cent permanent set	70	14,300,000 − 4.9 × S
485	Do., relief-annealed, 0.040 in. diam.	69	15,680,000 − 13.8 × S
494	Phosphor-bronze wire, 0.040 in. diam.	70	15,540,000 − 5.0 × S
	Do., after 0.16 per cent permanent set	70	15,380,000 − 4.5 × S

S = unit stress in tension, lb. per sq. in.

[1] The figures obtained using dead-weight loading and long gage length are somewhat more reliable than the other figures.

Due to the amount of elastic creep and recovery present, it is somewhat more difficult to specify true values for modulus of elasticity for the other materials. By varying the technique of operation, varying amounts of this elastic after-effect can be included in the readings, with resulting effect on the apparent modulus of elasticity. After some experimenting, it was found that reasonably consistent results could be obtained by taking the reading in each case at one minute after application of the load increment.

As obtained from the data so far obtained, the values of modulus of elasticity given in Table 1 are tentatively suggested. It should be emphasized that these values represent only the particular materials tested. A large amount of time and attention has been required for each series of tests, and therefore only a limited variety of samples have been tried. The investigation has not yet gone far enough to justify conclusions as to whether the rate of change of modulus of elasticity is a definite characteristic of the material or whether it varies with different types of heat treatment or of mechanical work. Further experiments are in progress for the purpose of covering some of these points.

A detailed study of the original data shows for each material a decrease of modulus of elasticity following overstresses great enough to produce a permanent elongation of the specimen. This is evident even for elongations no greater than one-tenth of 1 per cent, and even for material already severely cold-drawn. The per cent change in modulus is several times as great as the corresponding permanent set, even after all corrections have been made for the change in length and change in area of cross-section. There is a certain amount of evidence suggesting that in some cases with more severe overstressing there may be a reversal, the modulus increasing somewhat. A further knowledge of this factor is needed in order that we may understand the true stress conditions in cold-wound springs or in springs after surging and the investigation is being continued.

Discussion

R. L. TEMPLIN.[6] The author has given some very accurate and very interesting test results obtained from precise measurements of strain using very long gage lengths or specimens. The difficulties encountered in making such tests are quite numerous because factors which can frequently be ignored in the short-time tests may become of considerable significance in tests of the type represented.

It is generally accepted that at stresses near the elastic limit the modulus of elasticity of metals decreases quite rapidly, with the result that many investigators attempt to show that the stress-strain curve is curved from the origin. Dr. R. H. Canfield of the Naval Research Laboratories, Washington, D. C., pointed out before the Third International Congress, held at Stockholm last summer, that there appears to be a threshold stress below which the internal friction or slip was less than any measurable quantity. Interpreted in terms of Figs. 6 and 7 of the paper, this would mean that the curves would tend to become horizontal near the zero stress. Close scrutiny of the data presented by the author would appear to justify this conclusion. Fig. 7 shows data which exhibit this tendency to a marked degree, while there is less of a tendency in that direction shown in the data for Fig. 6.

R. W. CARSON.[7] The author is working on a problem which is interesting many manufacturers of precision apparatus. For rough-and-ready measurements of loads under the elastic limit, Hooke's law is usually satisfactory. However, when an accuracy greater than 1 or 2 per cent is required, these set and creep effects begin to manifest themselves. As the author indicates, the greater the sensitivity, the more involved becomes this elastic after-effect.

Is not the effect of moderate overstrain in cold-drawn phosphor-bronze wire to cause plastic flow in zones having high residual stresses, thus leaving the wire when returned to the unloaded condition with uniformly smaller residual stresses in the direction of the loading? If so, upon again moderately loading the wire in the same direction as that in which the over-stress was applied, the total of all the stresses across the section would then be smaller by an amount equal to the residual stresses relieved. Thus, the difference in elastic limit as noted in Fig. 7 would indicate the intensity of the residual stresses relieved rather than a change in the elastic properties of the material.

The same apparent change in modulus of elasticity might result from a mild heat treatment that would be just sufficient to allow a redistribution of high local stresses which would result in lower average stresses.

If the rate of creep or recovery action decreases with time and increases with stress, at some time very soon after the application or removal of a given load the creep or recovery action might equal the rate of purely elastic strain. Thus, the purely elastic and the creep or recovery action might be continuous and might be one and the same action over the entire time scale from the instant of applying load to the instant of further changing the load.

If such be the case, the width of the hysteresis loops would indicate only a relative amount of creep and recovery dependent on the time at which readings are taken. The recovery action might be of a slightly different order than the creep action, and in that event the reading time after applying load would vary for increasing or decreasing loads. Further, the hysteresis loop would not represent internal friction or work, since with sufficient time all of the work done in creep would be utilized in recovery.

W. G. BROMBACHER.[8] The fundamental nature of the author's work on the elastic behavior of spring materials should be emphasized. While the research does not now appear to have much application in many cases, undoubtedly, as with most fundamental research work, the results will be found to be use-

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ful. There is a tendency at the present time in metallurgical literature to avoid giving exact data on the moduli of elasticity. It is the practice to give either approximations or the two limiting values of the moduli. This is a clear recognition of the lack of precise knowledge as to the moduli of a given material under various conditions. The author's experiments are bringing light on this matter. His results show clearly that "elastic after-working" and "elastic hysteresis" must be considered in obtaining accurate and complete experimental data. These elastic effects have long been known and recognized by workers, especially those in the field of precision instruments. The first clear analysis of these phenomena was presented by Boltzmann, and it may be added that but very little improvement has been made up to the present in his theoretical presentation.

Special attention should be called to the author's method of experimentation since it is peculiarly well adapted for obtaining satisfactory data. It has two distinct advantages over most methods which should be clearly recognized. First, the specimen is uniformly stressed during the experiments. Second, the large length of the specimen, about 50 feet, permits results to be obtained to the essential high order of sensitivity with a minimum of expense and complication. The first advantage, of course, pertains only to the data obtained when the specimen is stressed in tension. The method will also enable data to be obtained for the torsional stresses, in which case the stressing will be non-uniform.

The lack of a satisfactory and universally accepted nomenclature should be stressed. Thus the author uses the term "elastic after-effect," which is referred to in the literature on the subject by such terms as "after-working," "drift" or "creep," and "recovery." These terms usually refer to the effect which is dependent on the time of application of the load as well as on the fiber stress and the material. The term "mechanical hysteresis" used in the report has been designated by other workers as "statical hysteresis," "elastic hysteresis," and "elastic lag due to elastic hysteresis." These terms usually refer to the effect which is independent of the time of the application of the load, but of course may depend on the fiber stress and the material. It should not be assumed that the preceding list of terms is complete, since many other expressions have been coined. It would appear desirable that workers in this field give consideration to obtaining the universal acceptance of a consistent nomenclature.

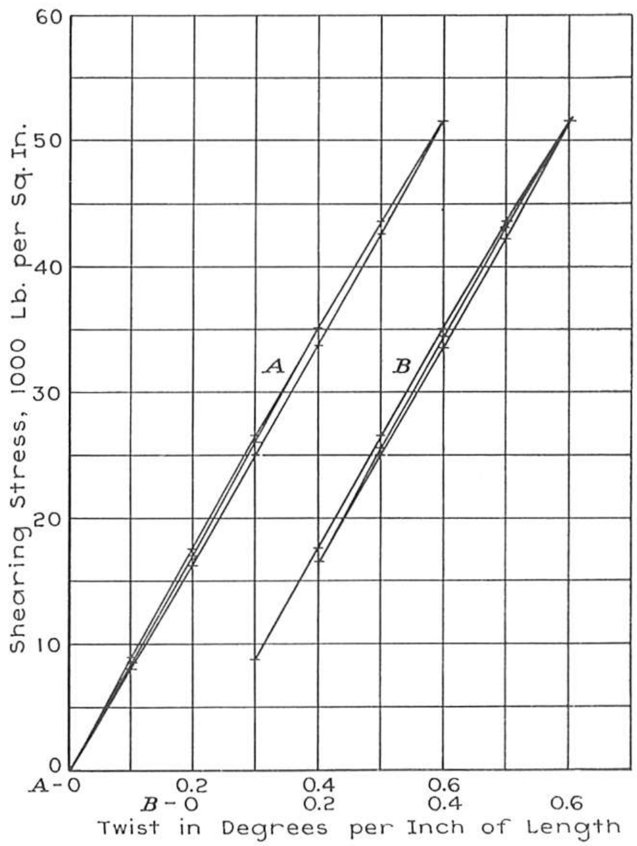

FIG. 8 HYSTERESIS CURVES—MILD STEEL

W. J. SWEETSER.[9] The author mentions the noticeable vibration set up in the test specimen when a truck passed at some distance from the building in which the work was being done. It seems probable that some of the apparently erratic behavior of the test specimen under constant load may be due to vibrations which are not detectable by ordinary means. The set-up of the test with suspended load is ideal for producing impact by vibration, and even unobserved vibrations of the foundation under the tank might give observable changes in elongation, especially when the order of magnitude of the measurements is so small.

Fig. 1 shows the curves for hysteresis and modulus-of-elasticity measurements, but in work which we have been doing at the University of Maine on steel in torsion, we have been unable to get such a series of readings as indicated at (*b*). Fig. 8 shows the hysteresis curve of an overstressed piece of mild steel in torsion. Three months before these curves were made the steel had been stressed to over 60,000 lb. per sq. in. in torsion and had been allowed to rest in the meantime. In curve *A*, the stress was increased gradually up to the maximum shown and then reduced to 0, giving a complete hysteresis loop. It was then restressed up to 35,000 lb. per sq. in. and returned to 0, giving the narrow hysteresis loop. The up curves in the two cases coincide. Curve *B*, shown at the right, shows a second hysteresis loop made by reducing the load from the maximum down to a stress of about 16,400 lb. per sq. in. and then returning to the maximum stress. In this case the curve coming down coincides with the original down side of the loop.

It is quite evident that the down side of the small loop in *A* is steeper than the down side of the larger loop, and similarly the up side of the small loop in *B* is steeper than the up side of the large loop, which would indicate that if a series of these small loops were put together as shown in (*b*), Fig. 1, the general slope of the series of small loops would not be the same as the mean slope of the curves shown at (*a*).

Taking the up curve of one of the small loops in (*b*), if it were continued it would tend to cross over and to the right of the coming down side of the loop above, and the writer is unable to see how this condition is possible.

D. E. ACKERMAN.[10] In his presentation of the paper the author mentioned that the curvature of the wire due to coiling on a drum was an important source of experimental error, and stated that several lots of wire had to be discarded because they had been coiled on too small a radius. It would be of interest to know the order of magnitude of the error, introduced in this way, as a function of the wire diameter and radius of curvature.

In Figs. 6 and 7 the author has indicated that the modulus-of-elasticity–tensile-stress relationship is linear. The writer would ask him whether the scattered data of curves 1 and 3,

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Fig. 7, are indicative of a high experimental error in these two cases, or whether the relationship for the corresponding materials simply follows a law of higher order; smooth curves can be drawn through the points plotted for the two curves in question. This point is of practical interest, for if the true relationship is of a higher order the plotted data suggest that at high tensile stresses the more severely cold-worked material may have the higher modulus of elasticity.

Author's Closure

One of our greatest difficulties in securing satisfactory results in these long-wire tests has been that of obtaining wire which was sufficiently straight for the purpose. The effect of local kinks or of curves in wire is, of course, greatest under light loads, the wire straightening out under heavier loads. As a result, there has been frequently found a spurious low modulus of elasticity under light loads. This has been very marked, indeed, in several cases, and unmistakable in its origin. It has therefore been necessary to discard altogether the results obtained from certain samples of wire and to use a grain of salt in interpreting other results where the author did not feel justified in discarding them entirely. Of the wires listed in Figs. 6 and 7, the steel ones were almost perfectly straight as received. The sample of aluminum wire which we were able to use was sufficiently straight so that it gave us very little trouble. The phosphor-bronze and brass wires were coiled on smaller-sized spools and so had a more pronounced curvature. This leads the author to regard with some suspicion the readings for these wires taken under light load.

On all the bona-fide straight samples which we have tested so far, our stress-strain line has been curved from the beginning. The one point of doubt remaining in the author's mind is as to the effect of elastic creep or elastic after-working upon the results. When a metal is loaded, most of the elongation occurs practically instantaneously, but it continues to elongate at a rapidly decreasing rate for five minutes or longer. This is true even at very low stresses. On removing the load an identical phenomenon occurs, but in the reverse direction. It is just possible that if our readings could be made coincidentally with the application of the load, and this after-effect entirely removed, our stress-strain line would be perfectly straight.

There are, however, very strong theoretical reasons for believing that this would not be so. For example, it has been definitely known for some time that the modulus of elasticity of a material under hydrostatic compression increases with increasing loads. The modulus in hydrostatic tension will obey the same equation as for hydrostatic compression, but with reversed sign, decreasing with increasing loads. Unidirectional tension is essentially a combination of hydrostatic tension and shear, and a direct relation should exist between the variation with stress of the moduli for hydrostatic tension and for direct tension. This is simply one of a series of theoretical reasons which make the author feel that it is highly unlikely that a stress-strain line for such relatively homogeneous materials as those which are here under investigation will be anything else but curved. Inhomogeneous materials such as wood, concrete, and cast iron also show a curved stress-strain line, but this curvature occurs for different reasons and is of an entirely different character, the modulus decreasing with increasing stress both in tension and in compression.

Reference may well be made to various comments on the erratic behavior shown by the plotted points in Fig. 7 for the 70–30 brass relief-annealed wire. A second wire of the same material, tested since December, has given the same curved line arrangement of points, so that it is apparently characteristic of the material. In certain of the other materials the decrease in modulus of elasticity at high stresses can be attributed to the permanent lowering of modulus which is likely to follow overstressing, but this does not appear to be the case in the wire just mentioned. No other material so far tested has duplicated this phenomenon, and the author does not as yet know how to explain it. It might, however, be well to notice that at low stresses the curved line which best represents the readings for wire No. 3 becomes asymptotic to the inclined line for specimen No. 1 rather than to a horizontal line.

The suggestion made by Mr. Carson that the creep or recovery action might be essentially continuous with the purely elastic action, is identical with the theories advanced by von Bennewitz in a paper in the *Physikalische Zeitschrift* for 1924. Von Bennewitz gave mathematical equations based on experiment to justify his theory, but these equations involved so much extrapolation beyond the limits of his experimental work that they have not been regarded with favor.

Figure 1(*b*) is misleading in that it shows the series of small loops as directly in line at their tips. Experimentally, when passing from cyclic condition for one load range to cyclic condition for a slightly higher load range, there is inevitably a certain amount of permanent, or semi-permanent, deformation so that two loops do not exactly match at their tips. This deformation will, the author believes, account for the discrepancies mentioned by Professor Sweetser as compared to his curves *A* and *B* in Fig. 8. When computing the modulus of elasticity, correction was made for this deformation so that figures were based on the effective slope of the individual small loops only.