Stochastic processes to model impact events in a vibratory cavitation erosion apparatus

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Abstract

Cavitation erosion is caused by bubble collapse causing micro jets and shockwave that impact the surfaces of materials: these impacts occur randomly over time. As such, Stochastic processes, specifically the Poisson process, can be used to model impacts as random events occurring over time. Peaks in pressure amplitude measurements made in time with a high-speed PVDF transducer in a vibratory cavitation erosion apparatus based on the ASTM G32 standard were defined as impact events. Using cumulative impact as a function of time, the rate parameter of the Poisson process was observed to vary periodically, which indicates that the process may be a non-homogeneous Poisson process (NHPP). Using a nonparametric estimation method, the NHPP's cumulative rate function was estimated, and then was fitted to a sinusoidal function with a frequency effectively equal to the vibratory apparatus' vibration and an average impact rate of 2.10×10^5 impacts per second.

Keywords: Impacts; Cavitation erosion; Vibratory apparatus; Poisson process; Stochastic process

Introduction

Cavitation occurrence in turbines, pumps and other turbomachines is a troublesome issue: cavitation bubble impacts are a major cause of erosion and mass loss itself is difficult to model accurately [1]. Significant discrepancies between modeled erosion rate and observed damage are still noted today [2]. One can realize the complexity and scope and the task undertaken: turbulent flow leading to nucleation, complex bubble-bubble interaction then violent collapse, forming shockwaves and micro-sized water jets, leading to deformation and erosion under high stress/strain rate [3].

Cavitation impact occurrence depends on parameters local to vapor bubbles that are difficult to measure [4,5], and cavitation bubble behavior is complex, a field of study on its own: bubble dynamics. It could prove useful to model such events as randomly occurring over time using stochastic processes, specifically Poisson point processes that can describe the occurrence of events in space and time [6-8]. From the point of view of the material, impacts also occur randomly on a 2D plane of generally flat geometry, and have other measurable characteristics: an amplitude, an area of effect, impact duration etc. that vary randomly. Here we will concentrate on modelling cavitation impacts events in time, and ignore other random variables. The confidence in the applicability of the Poisson process, the simplest of the point processes, to model cavitation impact events will be computed.

Body

The impact amplitude distribution was measured on a vibratory cavitation erosion apparatus inspired by the ASTM G32 standard [9]. The vibratory apparatus used had a vibration frequency of 19.5kHz and a peak-to-peak vibration amplitude that could be adjusted from 7μm to 20μm. A pump was used to recirculate water from the double-sided beaker to a water container: the tap water around the specimen was maintained at $25 \pm 5°C$. A schematic of the whole setup measurement setup is presented in Figure 1.

A Müller high-speed PVDF transducers model M60-1L-M3 was used: it has a rise time of 60ns, a sensitive diameter of 1mm, a sensitivity of -23.30 MPa/V and has a the following dynamic pressure range: -3MPa to 40MPa. A Tektronix MSO4054 oscilloscope was used to visualize and record the transducer output, with a sampling rate of at least 25MS/s, equivalent to a timestep of 40ns, close to the rise time. The event threshold was around 10mV equivalent to a minimum detected impact pressure of 200kPa, after filtering the data between 4kHz and 2MHz using Python.

The basic definition of a stochastic process is: a collection of random variables defined on a common probability space indexed by a set of numbers, generally time, which describes the system's evolution. In this context, the Poisson

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process is interpreted as a counting process [6-8]: we write $\{N(t), t \in T\}$, with T the continuous time $T = |0, \infty)$ and $N(t)$ the total number of impact events that occurred in the interval $[0, t]$. An impact event is defined as a local voltage/pressure maxima in the transducer data that exceeded the 10mV noise threshold.

The simplest Stochastic process that can be used to count events is the Poisson process [6]. It has several key properties, among which we cite: the number of events in a time interval Δt is completely independent from the number of events in all other intervals. It seems reasonable to assume that the impact events are independent, though some known physical mechanism demonstrate clustering behavior (a parent event leading to several children events). Bubble dynamic computations, confirmed by experiments, demonstrate that a single bubble collapse event can produce multiple predictable impact events because of multiple collapse-rebound cycles, under certain conditions [11]. Such repeated events can be taken into account by the so-called Cluster Poisson Process if they arise.

The Poisson process is generally defined using its probability density function (PDF), which outputs the probability P that the total number of events $N(t)$ in the interval $[0, t]$ is equal to i, noted $P[N(t) = i]$:

$$P[N(t) = i] = \frac{(\lambda t)^i}{i!} e^{-\lambda t}, \qquad t \geq 0 \tag{1}$$

With λ the single parameter of the Poisson process: the rate, or intensity. To compare the Poisson process with the recorded data, it is useful to learn how to how to estimate the PDF from observations, and how to draw samples using a PDF, or to simulate random variates. For the Poisson process, the expected number of points in a region is simply $E[N(t)] = \lambda$t. The strong law of large numbers leads to: $\lim_{t \to \infty} \left(\frac{N(t)}{t}\right) = \lambda$, which demonstrates that the best estimation method for the parameter of a Poisson process interpreted as a counting process is the total number of measured events divided by the total recording time.

As for sample generation, a simple method can be derived using the fact that the impact interarrival times are exponentially distributed with parameter λ: $P[\tau_i > t] = e^{-\lambda t}$. This function takes a time and outputs a probability: by isolating the time, we produce a function that outputs the time to the next event, as a function of a probability. Then, using a uniform random number generator a distribution of interarrival times can be obtained iteratively:

$$t_{next} = -\frac{\ln(u)}{\lambda} \tag{2}$$

with $u \sim U(0,1)$ the randomly generated probability. With this basic understanding of stochastic processes, we can start the analysis of the impact data. In Figure 2 (a), the raw pressure as a function of time is shown. There seems to be some periodicity: regions with high density of impacts repeat every 40-60μs at minimum, indicating it is the fundamental frequency of a periodic phenomenon, which implies a Poisson process may not be suitable to model such a phenomenon. By comparing the interarrival times of a Poisson process of rate equal to the total impacts divided by the record time, Figure 3 can be obtained. Cavitation impact events do not seem to follow a Poisson distribution: interarrival times are supposed to be exponentially distributed e.g. the number of long time periods with no impacts is too large. To take the apparent periodicity of the rate function into account, one can turn to the Non-Homogeneous Poisson process (NHPP)

The Poisson process described in Equation 1 indeed is homogeneous: the rate is constant in time. But one can obtain a NHPP by letting the rate function vary with time : $\lambda(t) \geq 0$. In the present case, the rate function is clearly periodic. The period of the pressure wave created by the apparatus is approximately 51.3μs, which provides a naïve explanation of this phenomenon: cavitation bubbles may increase in size in the low-pressure and collapse during the high-pressure period. Unfortunately, the bubble dynamics in a vibratory apparatus are complex [12] and outside the scope of the present research.

In any case, it is quite a bit more a bit more difficult to estimate the NHPP's rate function, but not at all impossible. The estimation of the cumulative rate function by the nonparametric method proposed by Leemis [13] was used. The cumulative rate function of a NHPP, which is its expectation function $\Lambda(t) \equiv E[N(t)]$, is defined [6-8]:

$$\Lambda(t) = \int_0^t \lambda(\tau) d\tau, \qquad t > 0 \tag{3}$$

The Leemis estimator gives an approximation of Λ, which is noted $\widehat{\Lambda}$, for a number k of realizations of the observation of the NHPP in a time window $(0, S]$. S can be, for example, all opening hours (e.g. 9AM to 5PM) of a restaurant that wishes to count the arrival of customers. The estimator is written:

$$\widehat{\Lambda}(t) = \frac{in}{(n+1)k} + \left[\frac{n(t-t_i)}{k(n+1)(t_{i+1}-t_i)}\right], \qquad t_i < t \leq t_{i+1};\ i = 0,1,2,\dots,n \tag{4}$$

$\widehat{\Lambda}$ is the estimation of Λ, n is the total number of impacts recorded in k realizations, t_i are the sorted arrival times of the n impacts and t represents an arbitrary large number of times between t_i and t_{i+1}. In this case, S was chosen to be long enough to include at least 10 periods: $S = 4ms$, for $k = 100$ realizations. The value of the estimator is linear between, t_i and t_{i+1}. If two or more impact have the same arrival time t_m, the estimator jumps to the value:

$$\widehat{\Lambda}(t_m) = \widehat{\Lambda}(t_{m+1}) = \frac{mn}{k(n+1)} \tag{5}$$

Using this estimator, the rate function could be fitted to a function of the form: $\lambda(t) = A(\sin(2\pi\nu t + \phi) + 1)$ (see Equation (3)). In the present case, for a distance of 1.4mm between the vibrating and head and the sensor and a vibration amplitude of 7µm, $A = 2.10 \times 10^5$ impacts per second. Also, the estimated period is 50.6µs, close to the expected 51.3µs. The frequency of the rate function (19.8kHz) is quite similar to the vibratory apparatus' (19.5kHz), but it is not possible to comment further without additional impact data, and studies on the bubble dynamics.

In Figure 4 are presented the results of the estimation $\widehat{\Lambda}$ compared with the fitted sinusoidal function, with 95% confidence intervals (CI). One must take heed as the CI in Figure 4 (a) are different from those in Figure 4 (b): in Figure 4 (a) the CI are on the expected number of events given the rate of a Poisson process (one can learn to compute such intervals using the Poisson distribution such as described in [16] p. 182 for example), in Figure 4 (b) the CI are on the estimation $\widehat{\Lambda}$ given the observed data, as described by Leemis [13]. Figure 4 (c) presents the error between the estimation $\widehat{\Lambda}$ and the fitted sinusoidal function, compared with the error between the estimation and its CI bounds. One expects the cumulative function to be contained in the CI bounds in 95% cases: the current fit is outside these bounds in 8.98% cases, which is high. If the current estimation $\widehat{\Lambda}$ is correct, 31.6% of all measured points are out of the confidence bounds of the NHPP, as can be observed on the data in Figure 4 (a). The authors feel the experimental setup may explain this unusually high variance. For example, local variation of temperature in the cavitation region may be due to the vibrating head acting as a heat source: temperature was observed to rise close to the sample even with temperature control using a double-sided beaker, because of the long time data recording necessitated (2min by realization). A higher variance could be explained by the rate function $\lambda(t)$ itself varying over time. Better control could provide clearer insight. Another explanation may be that some parameter in the rate function varies randomly:the local temperature may oscillate randomly between realizations and influence the rate. In such a case, the Mixed Poisson process could be used to model the impact events. If the rate function itself is a stochastic process, then the cavitation impacts events can be modelled using the doubly stochastic Poisson process (Cox process).

This analysis was proposed as the first step in the elaboration of a stochastic process that could capture the random nature of the cavitation erosion process. In subsequent experiments, the vibration frequency, vibration amplitude, water temperature among others should be changed to observe the effect on the rate function. Also, steps will be taken to model events in the vibratory apparatus using the mentioned other stochastic processes. Such stochastic processes could be used to model cavitation impact events in turbines and pump as well as cavitation erosion laboratory apparatuses such as the cavitation impact jet described in the ASTM G134 standard [17]. They could also prove useful to build a mass-loss prediction model, by combining them with deformation and failure under strain hardening and high strain-rate conditions, such as the Johnson-Cook dynamic failure model [10].

Figures & Tables:

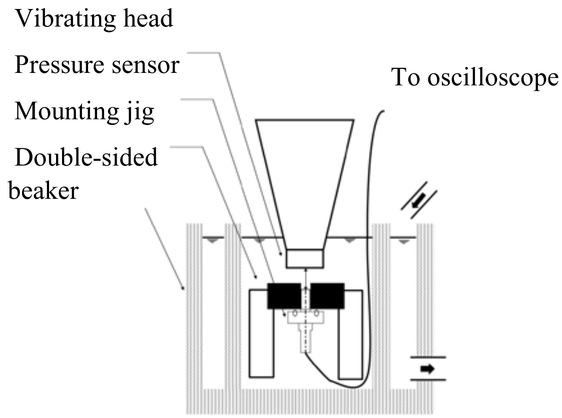
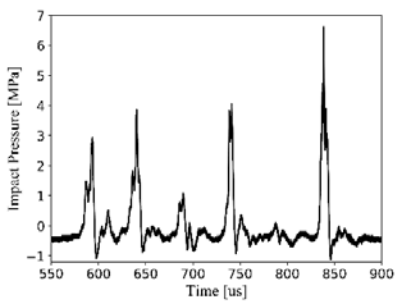
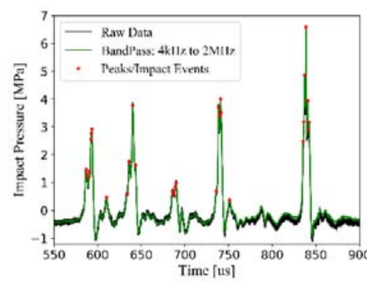


Figure 1: Vibratory apparatus setup to record impact pressure as a function of time



(a): Raw high pressure sensor output



(b): Peaks were detected taking the first order difference (with peakutils, a python package)

Figure 2: Impact pressure as a function of time for 1.4mm distance and 7µm amplitude

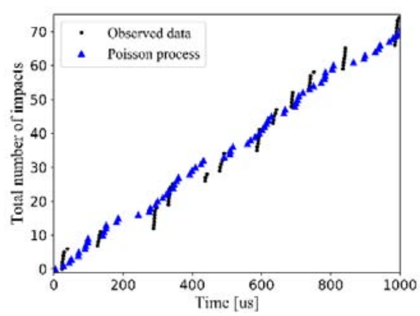
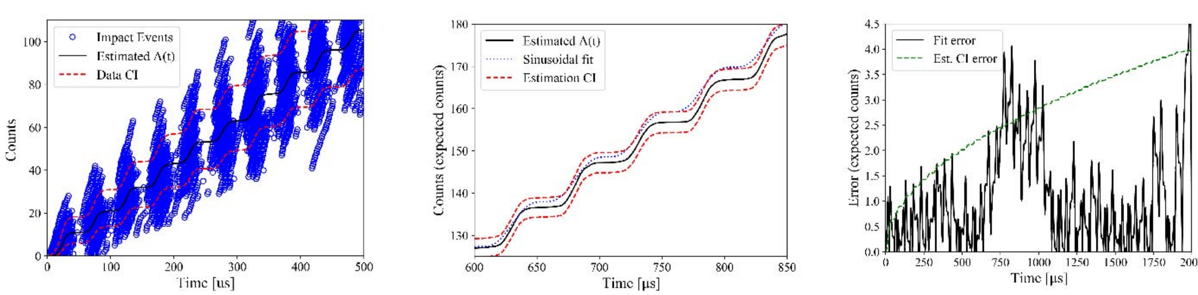


Figure 3: Cumulative events as a function of time: Comparison with Poisson process of equivalent rate



(a): Observed data for $k = 100$ realizations compared with $\widehat{\Lambda}$ estimation

(b): Estimated rate function fitted with a sinusoidal rate function

(c): Fitting error: 8,98% of points of the fit are outside the 95% CI

Figure 4: Rate function estimated using the nonparametric method in Leemis [13]

Conclusion

Cavitation erosion impacts occur randomly over time and as such can be modeled as a Stochastic process. The simplest is the Poisson process, for which the only parameter is the rate parameter. Using a high-speed PVDF pressure sensor to detect impacts in a vibratory apparatus, periodic spikes in the arrival indicated that the rate parameter varies over time: the Poisson process is non-homogeneous. Using a non-parametric method by Leemis [13], the cumulative rate function was observed to be sinusoidal, with a frequency close to the driving transducer: 19.8kHz compared to 19.5kHz. Cavitation impacts were recorded for 4ms, 100 times to produce said estimation. An average of 2.10×10^5 impacts per second were observed. Fitting the estimated cumulative rate function using a sinusoidal function proved relatively accurate, but 31.8% of all data points fall outside the 95% confidence intervals which is unusually high.

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