A Reduced Order Gas Pressure Law for Single Acoustic Cavitation Bubbles

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Abstract

An acoustic cavitation model for noncondensable gas/vapor bubbles that couples spherical bubble dynamics by the Keller-Miksis equation to the Plesset-Zwick equation is constructed by accounting for phase change, but neglecting the mass diffusion of the noncondensable gas. Results obtained for acoustically driven air/water-vapor cavitation bubbles using two different acoustic pressure signals and variable fluid properties show reasonable agreement with the reduced order model of Preston et al.

Keywords: acoustic cavitation, reduced order gas pressure law, bubble dynamics.

Introduction

Acoustic cavitation has aplications in sonochemistry [1], sonoluminescence [2,3] and medical ultasound [4,5] where bubbles of a few microns size grow and collapse to sizes that vary by orders of magnitude, especially at ultrasonic frequencies. In this case the pressure and temperature inside the bubble can show variations by orders of magnitude. Many complex acoustic cavitation models are constructed [6-12] for the numerical simulation of the temperature and pressure distributions under acoustically driven frequencies. However, when the complexity of the models and the computational time required for their simulation are taken into account, the need for simplified expressions for the gas pressure and temperature inside the bubble is obvious. This need becomes more important in hydrodynamic cavitation. In this investigation the heat conduction through the bubble is considered by the solution of Prosperetti's equation [8] in the uniform pressure approximation by a novel iterative method [13]. This iterative method leads to the desired reduced order gas pressure law exhibiting power law dependence on the bubble wall temperature and bubble radius, with the polytropic index depending on the isentropic exponent of the gas and on a parameter which is a function of the Peclet number. Moreover, it is shown that this reduced order gas pressure law reduces to the classical isothermal and adiabatic laws in the appropriate limits of the parameter. The bubble wall temperature entering this reduced order gas pressure law is obtained from the Plesset-Zwick solution [14]. Using this reduced order gas pressure law, an acoustic cavitation model for noncondensable gas/vapor bubbles that couples spherical bubble dynamics by the Keller-Miksis equation to the Plesset-Zwick equation is constructed by accounting for phase change, but neglecting the mass diffusion of the noncondensable gas. Results obtained for acoustically driven air/water-vapor cavitation bubbles using two different acoustic pressure signals and variable fluid properties show reasonable agreement with the reduced order model of Preston et al. [10] for a suitable value of the parameter.

An Acoustic Cavitation Model Using A Novel Reduced Order Gas Pressure Law

We consider the thermal behavior of a single spherical gas bubble surrounded by a liquid in the uniform pressure appproximation. The temperature distribution inside the bubble is then given by the Prosperetti equation [8, 13,15]

$$\frac{p}{T}\left\{\frac{\partial T}{\partial t}+\frac{1}{(Pe)\,pR^2}\left[\lambda(T)\frac{\partial T}{\partial y}-y\left(\frac{\partial T}{\partial y}\right)_{y=1^-}\right]\frac{\partial T}{\partial y}\right\}=\frac{(\gamma-1)}{\gamma}\frac{dp}{dt}+\frac{1}{(Pe)R^2y^2}\frac{\partial}{\partial y}\left[y^2\lambda(T)\frac{\partial T}{\partial y}\right] \qquad (1)$$

where T is the temperature within the bubble, p is the uniform gas pressure inside the bubble, R is the instantaneous bubble radius, $\lambda(T)$ is the temperature dependent thermal conductivity of the gas, γ is the isentropic exponent of the gas, y is the normalized radial coordinate normalized and t is the time, all normalized with respect to some reference quantities [15]. In eq. (1) the Peclet number Pe is defined by

$$Pe=\frac{\gamma p_0' R_0'^2}{(\gamma-1)\lambda_R' T_0' \Theta'} \qquad (2)$$

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where p'_0 and T'_0 are, respectively, the bulk liquid equilibrium pressure and temperature with Θ' denoting a characteristic time, λ'_R denoting the thermal conductivity of the gas at the bubble wall temperature and R'_0 denoting the initial equilibrium bubble radius. The gas pressure within the bubble in the uniform pressure approximation is then obtained by solving the differential equation

$$\frac{dp}{dt}=\frac{3\gamma}{R}\left[\frac{1}{(Pe)R}\left(\frac{\partial T}{\partial y}\right)_{y=1^-}-p\frac{dR}{dt}\right]. \tag{3}$$

The system of the coupled differential eqs. (1) and (3) is then solved subject to the initial and boundary conditions

$$p(0)=p_i \quad ; \quad T\left(t=0,y\right)=1 \quad ; \quad T(t,y=1^-)=T_R(t) \quad ; \quad (\partial T/\partial y)_{y=0}=0 . \tag{4}$$

By expanding the temperature distribution near the bubble wall in the form

$$T(t,y)=T_R(t)+\left(\frac{\partial T}{\partial y}\right)_{y=1^-}\left(y-1^-\right)+\frac{1}{2!}\left(\frac{\partial^2 T}{\partial y^2}\right)_{y=1^-}\left(y-1^-\right)^2+O[(y-1^-)^3] \tag{5}$$

and by neglecting the temperature dependence of the gas thermal conductivity, substitution of eq. (5) into eq. (1) and taking the limit as $y\rightarrow 1^-$ yield the expression

$$c(t)=\left(\frac{\partial T}{\partial y}\right)_{y=1^-}=\frac{(Pe)R^2}{2}\left[\frac{p}{T_R}\frac{dT_R}{dt}-\frac{(\gamma-1)}{\gamma}\frac{dp}{dt}\right]-\frac{1}{2}\left(\frac{\partial^2 T}{\partial y^2}\right)_{y=1^-} \tag{6}$$

for the temperature gradient at the bubble wall. Utilizing eq. (6), we can obtain successive iterative approximations for the gas pressure from the solution of eq. (3). The first approximation, which neglects the second order radial derivative, is already discussed in [13], but this approximation does not produce dependence on the Peclet number and is thus inconvenient for all Peclet numbers. In the second approximation the second order radial derivative is taken to be proportional to the radial temperature gradient at the bubble wall, with the proportionality factor f exhibiting Peclet number dependence as well as weak dependence on time scale. Under this assumption substitution of eq. (6) into eq. (3), upon exact integration, leads to the novel reduced order gas pressure law in the form

$$p=p_i\left[\frac{\left(T_R\right)^{1/2(1+f)}}{R}\right]^{3\Gamma} \tag{7}$$

where Γ is a polytropic index given by

$$\Gamma=\frac{2\gamma(1+f)}{\left(3\gamma-1+2f\right)} \tag{8}$$

with f denoting a parameter depending on the Peclet number. It can further be shown that the novel reduced order gas pressure law given by eqs. (7) and (8) reduces to the isothermal law when γ=1 (or f=1/2) and T_R=1, and to the classical adiabatic law as $f\rightarrow\infty$. Using this reduced order gas pressure law, we can construct an acoustic cavitation model for bubbly liquids containing noncondensable gas/vapor bubbles. Assuming that the noncondensable gas/vapor bubbles consist of ideal gases and using the reduced order gas pressure law for the noncondensable gas, the total normalized mixture pressure p_b inside the bubble can be written in the form

$$p_b=p_v+p_g=p_{v,sat}(T_R)+p_{g0}\left[\frac{\left(T_R\right)^{1/2(1+f)}}{R}\right]^{3\Gamma} \tag{9}$$

where $p_{v,sat}(T_R)$ is the normalized saturation pressure of the vapor at the bubble wall and p_{g0} is the normalized initial equilibrium gas pressure. For the bubble wall temperature T_R, by assuming that most of the latent heat of condensation at the bubble wall is transferred to the liquid side since the thermal conductivity of the noncondensable gas/vapor mixture is much smaller than that of the liquid, we use the Plesset-Zwick solution in the form [14]

$$T_R(t) = 1 - B\int_0^t \frac{L(\xi)\rho_{v,sat}(\xi)R^2(\xi)(dR/d\xi)}{[\int_\xi^t R^4(\tau)d\tau]^{1/2}}d\xi \tag{10}$$

where $L=L'/L_0'$ and $\rho_{v,sat}=\rho'_{v,sat}/\rho'_{v0}$ with L' and $\rho'_{v,sat}$ denoting, respectively, the latent heat of vaporization of the liquid and the saturated vapor density at the bubble wall temperature and where 0 denotes the properties at the bulk liquid temperature. The constant B in eq. (10) is defined by

$$B = \left(\frac{L_0'}{T_0'c'_{p_l\,0}}\right)\left(\frac{\rho'_{v0}}{\rho'_{l\,0}}\right)\frac{R_0'}{\left(\pi\ \Theta'\alpha'_{l\,0}\right)^{1/2}} \tag{11}$$

with $c_p'_{\ell 0}$, $\rho'_{\ell 0}$ and $\alpha'_{\ell 0}$ denoting, respectively, the specific heat of the liquid, the density of the liquid and the thermal diffusivity of the liquid at the bulk liquid temperature. Using the reduced order gas pressure law for the partial pressure of the noncondensible gas, the spherical dynamics of the bubble can be characterized by the Keller-Miksis equation which takes the normalized form

$$\begin{aligned}
&\left[1 - MR + \frac{4M}{R(\mathrm{Re})}\right]R\ddot{R} + \frac{3}{2}\left[1 - \frac{M}{3}\dot{R} - \frac{8M}{3R(\mathrm{Re})}\right]\dot{R}^2 + \left[\frac{4(1+M\dot{R})}{\mathrm{Re}} - \frac{2M}{We}\right]\frac{\dot{R}}{R} \\
&+2(1+M\dot{R})\left\{\frac{1}{R(We)} - \frac{1}{We_0}\left[\frac{(T_R)^{1/2(1+f)}}{R}\right]^{3\Gamma}\right\} + \frac{(1+M\dot{R})}{2}\left\{\sigma - \sigma_0\left[\frac{(T_R)^{1/2(1+f)}}{R}\right]^{3\Gamma}\right\} \\
&+\frac{(1+M\dot{R})C_p}{2} - MR\frac{d}{dt}\left\{\left(\frac{2}{We_0} + \frac{\sigma_0}{2}\right)\left[\frac{(T_R)^{1/2(1+f)}}{R}\right]^{3\Gamma} - \frac{C_p}{2}\right\} = 0
\end{aligned} \tag{12}$$

where the variable cavitation number σ, the variable Reynolds number *Re* and the variable Weber number *We* are defined by

$$\sigma = \frac{p_0' - p'_{v,sat}}{\frac{1}{2}\rho'_{l\,0}U_0'^2}\ , \qquad \mathrm{Re} = \frac{\rho'_{l\,0}U_0'R_0'}{\mu_l'}, \qquad \text{and} \qquad We = \frac{\rho'_{l\,0}U_0'^2R_0'}{S'} \tag{13}$$

with S' denoting the surface tension, μ'_ℓ denoting the liquid dynamic viscosity (both to be evaluated at the bubble wall temperature), $U'_0 = R'_0/\Theta'$ denoting a characteristic speed and with subscript 0 denoting the initial value of the variable properties. The Mach number is given by $M=U'_0/c'_{\ell 0}$ with $c'_{\ell 0}$ denoting the speed of sound in the liquid, and the pressure coefficient entering the normalized Keller-Miksis equation (12) is defined as

$$C_p(t) = -\frac{p_A'(t)}{\frac{1}{2}\rho'_{l\,0}U_0'^2} \tag{14}$$

where $p'_A(t)$ is the deriving acoustic pressure. Equations (10) and (12) form the basic equations of the proposed acoustic cavitation model using the novel reduced order gas pressure law. For the numerical solution of the initial value problem of the Keller-Miksis equation (12) for spherical bubble dynamics, we use the Runge-Kutta-Fehler method with adaptive time step size together with the Plesset-Zwick solution (10) for the bubble wall temperature evaluated by Simpson's 3/8-numerical integration scheme iteratively. We now apply the proposed acoustic cavitation model to water-vapor/air bubbles in water. We choose two different acoustic deriving pressure signals with pressure coefficients given by [10,16]

$$C_p(t) = -0.25\left[1 - \cos\left(\frac{2\pi t}{500}\right)\right] \quad ; \qquad 0 < t < 500 \tag{15}$$

and

$$C_p(t) = -0.662\exp\left[-\left(\frac{t-130}{43.5}\right)^2\right]; \quad 0 < t < 300. \tag{16}$$

The bulk water temperature is chosen to be T_0'=20ºC. In this case the saturated vapor pressure is p'_{v0}=0.0234 bar, the surface tension is S_0'=0.071 N/m and the dynamic viscosity of water is $\mu'_{\ell 0}=10^{-3}$ kg/m-s at the bulk liquid temperature. For the acoustic driving pressure signal [16] given by eq. (15) , the initial cavitation number is σ = 0.492, the initial bubble radius is R_0'=100 μm and the characteristic time corresponding to the acoustic deriving pressure is taken as $\Theta' = 10^{-5}$ s which yield an initial Reynolds number *Re=1000* and an initial Weber number *We=137*. Figures 1(a) and (1b) show, respectively, the variations of the normalized bubble radius and of the normalized bubble wall temperature with time for different values of the parameter f (f=0.5, 1.0 and 10.0) using the present acoustic cavitation model. The time variation of the bubble radius shows reduction in the bubble oscillation amplitudes as the parameter f increases from 0.5 to 10.0 exhibiting typical behavior from near-isothermal case (low Peclet number) to near-adiabatic case (high Peclet number). The bubble wall temperature variations also show reduction in magnitude as the parameter increases from 0.5 to 10.0. For the acoustic driving pressure signal [10] given by eq. (16) , the initial cavitation number is σ = 0.654, the initial bubble radius is R_0'=100 μm and the characteristic time corresponding to the acoustic deriving pressure is taken as $\Theta' = 2.3 \text{ X } 10^{-6}$ s which yield an initial Reynolds number *Re=695* and an initial Weber number *We=167*. Figures 2(a) and (2b) show, respectively, the variations of the normalized bubble radius and of the normalized bubble wall temperature with time for different values of the parameter f (f = 0.5, 1.0 and 10.0) for this case using the present acoustic cavitation model. Similar behavior in the reduction of the bubble oscillation amplitudes and in the bubble wall temperature is observed as the parameter f increases from 0.5 to 10.0. In particular, the results for f =0.5 are in agreement with those given by Preston et al. [10] using their reduced order constant transfer model without mass diffusion where they employ the Rayleigh-Plesset equation instead of the Keller-Miksis equation. Finally, under the same conditions of Figure 2, we investigate the effect of variable fluid properties using the proposed acoustic cavitation model. Figures 3(a) and 3(b) show, respectively, the variations of the normalized bubble radius and of the normalized bubble wall temperature with time for variable fluid properties. Considerable reductions in both buble oscillatiion amplitudes and in bubble wall temperatures are observed as compared to the case of constant fluid properties. Moreover, a fast relaxation to equilibrium of the bubble radius and of the bubble wall temperature is observed depending on the value of the parameter f .

(a) **(b**

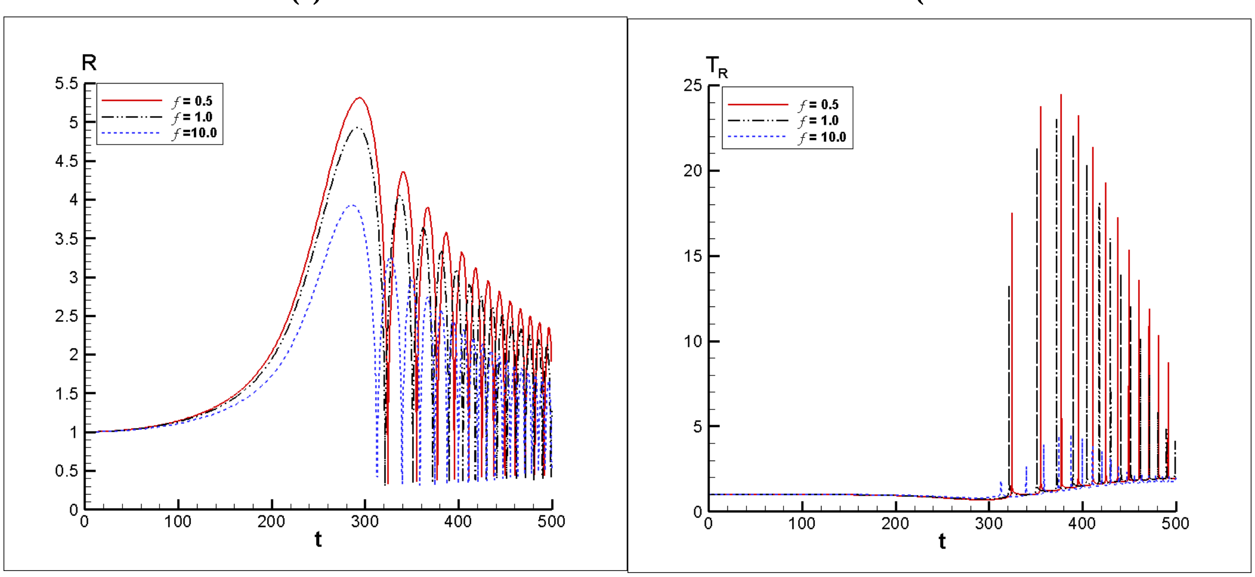

Figure 1: The temporal evolution of (a) the normalized bubble radius, (b) the normalized bubble wall temperature, driven by the acoustic pressure given by eq. (15) for an air-water vapor bubble in water with σ = 0.492, We=137, Re=1000 for different values of f (0.5, 1.0 and 10.0) obtained by the proposed acoustic cavitation model using constant fluid properties.

(a) **(b)**

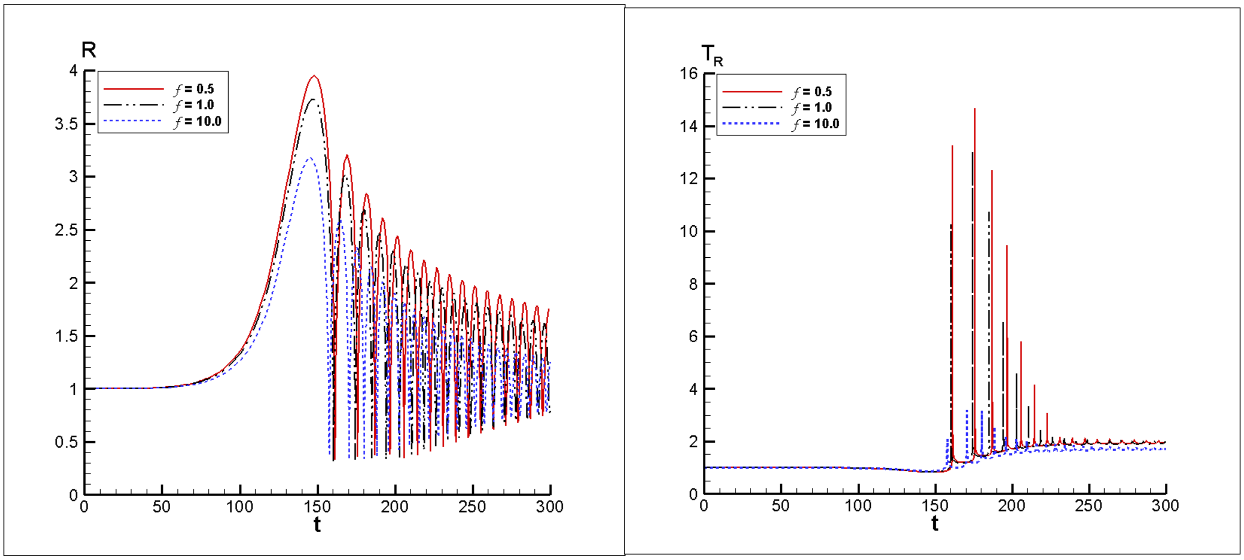

Figure 2: The temporal evolution of (a) the normalized bubble radius, (b) the normalized bubble wall temperature, driven by the acoustic pressure given by eq. (16) for an air-water vapor bubble in water with $\sigma = 0.654$, We=167, Re=695 for different values of f (0.5, 1.0 and 10.0) obtained by the proposed acoustic cavitation model using constant fluid properties.

(a) **(b)**

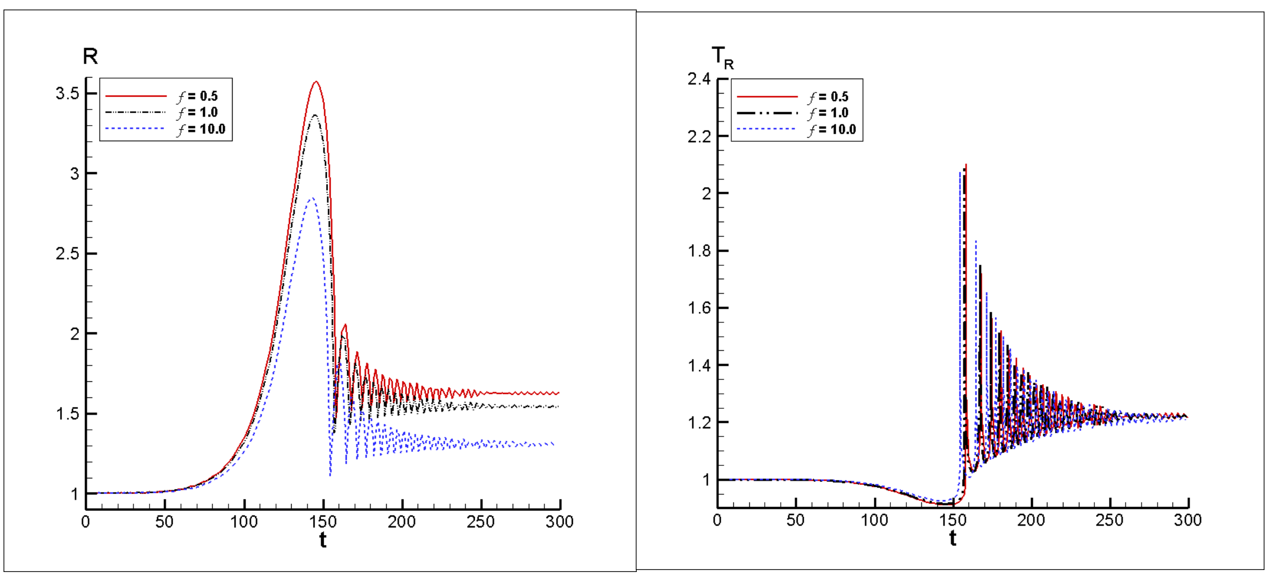

Figure 3: The temporal evolution of **(a)** the normalized bubble radius, **(b)** the normalized bubble wall temperature, driven by the acoustic pressure given by eq. (16) for an air-water vapor bubble in water with initial values $\sigma = 0.654$, We=167, Re=695 for different values of f (0.5, 1.0 and 10.0) obtained by the proposed acoustic cavitation model using variable fluid properties.

Conclusion

The thermal behavior of gas and gas/vapor bubbles is investigated by considering th energy balance between a single spherical bubble and the surrounding liquid, neglecting the effect of gas diffusion to the liquid. For gas bubbles we solve the well-known coupled differential equations for the gas pressure and temperature inside the bubble in the uniform pressure approximation by using an iterative technique for approximating the second order radial derivative of the temperature at the bubble wall. In this way we obtain a novel reduced order gas pressure law which exhibits power law dependence on the bubble wall temperature and bubble radius, with the polytropic index depending on the isentropic exponent of the gas and on a parameter, which is a function of the Peclet number and of the characteristic time scale. Moreover, it is shown that this parameter is an increasing function of the Peclet number where the reduced order gas pressure law reduces to the classical isothermal law when this parameter takes the value 1/2 and to the classical adiabatic law when this parameter tends to infinity. The bubble wall temperature entering this reduced order gas pressure law is obtained from the Plesset-Zwick solution of the temperature distribution of the liquid side in the thin boundary layer approximation. Using this reduced order gas pressure law, a reduced order acoustic cavitation model for noncondensable gas/vapor bubbles that couples spherical bubble dynamics by the Keller-Miksis equation to the Plesset-Zwick equation is constructed by accounting for phase change, but neglecting the mass diffusion of the noncondensable gas. Results obtained for acoustically driven air/water vapor cavitation bubbles using two different acoustic pressure signals and variable fluid properties show reasonable agreement with the reduced order model of Preston et al. [10] for a suitable value of the parameter. It remains to correlate the model parameter with Peclet number and characteristic time scale (driving frequency) using the results of the DNS of the original PDEs over a wide range of Peclet numbers and driving frequencies.

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