Journal of Applied Mechanics

Discussion

Discussion: "Combinations for the Free-Vibration Behavior of Anisotropic Rectangular Plates Under General Edge Conditions" (Narita, Y., 2000, ASME J. Appl. Mech., 67, pp. 568–573)

C. W. Bert
Perkinson Chair Professor, School of Aerospace and Mechanical Engineering, The University of Oklahoma, Norman, OK 73019-1052. Life Fellow ASME

The author is to be commended for his new approach to the important problem of calculation of natural frequencies for anisotropic plates. However, two comments are in order.

First, the paper lists only three classical boundary conditions: simply supported, clamped, and free. Actually, there is a fourth one: guided or sliding (Bert and Malik [1]). For this boundary condition, the effective shear force and the bending slope are both zero. Exact natural frequency results were given for a variety of such cases of isotropic plates in [1].

The second comment is that, although the Ritz method is an upper bound solution, it converges rather slowly in the case of anisotropic plates. For design purposes, a lower bound to a frequency is often more important than an upper bound. The convergence of the Ritz method, a Fourier series method (Whitney [2]), and the differential quadrature method were studied by Bert et al. [3] for free vibration of simply supported plates of highly anisotropic material ($E_L/E_T=25$, compared to 15.4 in the present paper). The latter two methods provided lower bounds.

References

[1] Bert, C. W., and Malik, M., 1994, "Frequency Equations and Modes of Free Vibrations of Rectangular Plates With Various Edge Conditions," Proc. Inst. Mech. Eng., Part C: J. Mech. Eng. Sci. **208C**, pp. 307–319.
[2] Whitney, J. M., "Free Vibrations of Anisotropic Rectangular Plates," 1972, J. Acoust. Soc. Am. **52**, pp. 448–449.
[3] Bert, C. W., Wang, X., and Striz, A. G., 1994, "Convergence of the DQ Method in the Analysis of Anisotropic Plates," J. Sound Vib. **170**, pp. 140–144.

Closure to "Discussion of 'Combinations for the Free-Vibration Behavior of Anisotropic Rectangular Plates Under General Edge Conditions'" (2001, ASME J. Appl. Mech. 68, p. 685)

Y. Narita
Department of Mechanical Engineering, Hokkaido Institute of Technology, 7-15 Maeda, Teine-ku, Sapporo, 006-8585, Japan
e-mail: narita@hit.ac.jp

Although the main idea of the paper is an introduction of the Polya counting theory to an engineering counting problem that may be encountered in applied mechanics, this author equally appreciates the interest shown by Professor Bert in the proposed Ritz method to calculate natural frequencies of anisotropic plates with arbitrary boundary conditions. Professor Bert raised two constructive comments that are answered in order.

The first comment is that the present author took up only three classical boundary conditions (i.e., free, simply supported, and clamped edges) in numerical examples and did not consider the fourth boundary condition of a guided or sliding edge with zero effective shear force and bending moment. The function (20) in the x and y-direction ([1]) is not applicable in its direct form to the fourth boundary condition but it is widely accepted that the fourth condition is not as important as the first three ones. It may be possible to apply the present function to the fourth boundary condition by adding a constant term to give a constant displacement caused by the guided or sliding edge and also constraining the slope at the edge.

The second comment, which is more important, is on convergence rates of the present solution applied to anisotropic plates. Before commenting on that, I have to make it clear that the caption in Table 2 of [1] was erroneous. The convergence result in the table was for a specially orthotropic square plate. (i.e., anisotropic plate with a fiber orientation angle $\theta=0$ deg), not for skew orthotropic square plates ($\theta=30$ deg). This was obvious that the converged values in Table 2 are in exact agreement with those of specially orthotropic plates in Table 5 of [1].

Professor Bert stated that "it (Ritz method) converges rather slowly in the case of anisotropic plates." I agree that the Ritz method tends to give slower convergence for anisotropic plates than for isotropic and specially orthotropic plates, but this tendency is also found for other methods and the important question is how slow the solution becomes. To see this, another test is conducted here to observe convergence rates of the present method for highly anisotropic material.

$$Q_{11}/Q_{22}=25,\quad Q_{12}/Q_{22}=0.25 \quad \text{and} \quad Q_{66}/Q_{22}=0.5$$

Table 1 Convergence of frequency parameters Ω of diagonally orthotropic square plates (θ=45 deg, Q_{11}/Q_{22}=25, Q_{12}/Q_{22}=0.25, and Q_{66}/Q_{22}=0.5)

B.C.	Num.of terms	1st	2nd	3rd	4th
FFFF	6x6	18.90	24.70	43.30	59.60
	8x8	18.82	24.60	42.67	58.79
	10x10	18.82	24.60	42.65	58.71
	12x12	18.82	24.60	42.65	58.70
	14x14	18.83	24.60	42.65	58.70
SSSS	4x4	53.87	95.06	165.5	178.1
	6x6	52.91	91.58	143.6	156.0
	8x8	52.46	91.55	142.4	154.4
	10x10	52.19	91.54	142.3	153.8
	12x12	52.01	91.54	142.3	153.3
	14x14	51.89	91.54	142.3	153.0
	16x16	51.80	91.54	142.2	152.8
CCCC	6x6	91.31	145.8	208.2	217.0
	8x8	91.24	145.8	208.0	216.3
	10x10	91.22	145.8	208.0	216.2
	12x12	91.22	145.8	208.0	216.2
	14x14	91.22	145.8	208.0	216.2
CFFF	6x6	5.947	18.28	35.78	43.23
	8x8	5.919	18.11	35.25	42.93
	10x10	5.904	18.05	35.12	42.90
	12x12	5.898	18.03	35.08	42.89
	14x14	5.898	18.03	35.08	42.89

used in reference [2]. This material has stronger anisotropy (E_L/E_T=25) than that (E_L/E_T=15.4) used in ([1]).

Table 1 presents convergence test results of diagonally orthotropic square plates (θ=45 deg) with such material for boundary conditions of FFFF (free plate). SSSS (simply supported plate), CCCC (clamped plate), and CFFF (cantilever plate). Frequency parameters

$$\Omega=\omega a^2(\rho h/D_0)^{1/2} \quad \text{with a reference stiffness } D_0 = E_T h^2/12(1-v_{LT}v_{TL}) \tag{1}$$

are presented with four significant figures for the number of terms $M\times N=6\times 6\sim 14\times 14$ ($4\times 4\sim 16\times 16$ for SSSS) in Eq. (15) of [1], and underlined figures are converged values within the range of our significant figures. It is seen that two extreme cases of the FFFF (totally free plate with only natural boundary conditions) and the CCCC (the most constrained plate with only geometrical boundary conditions) plates do yield fast convergence, while the SSSS plate with both natural and geometrical boundary conditions does slower convergence, particularly for the fundamental mode.

The convergence behaviors of some different solutions are compared for the fundamental frequency of the SSSS plate in reference [2]. The present values are converted to their frequency parameter

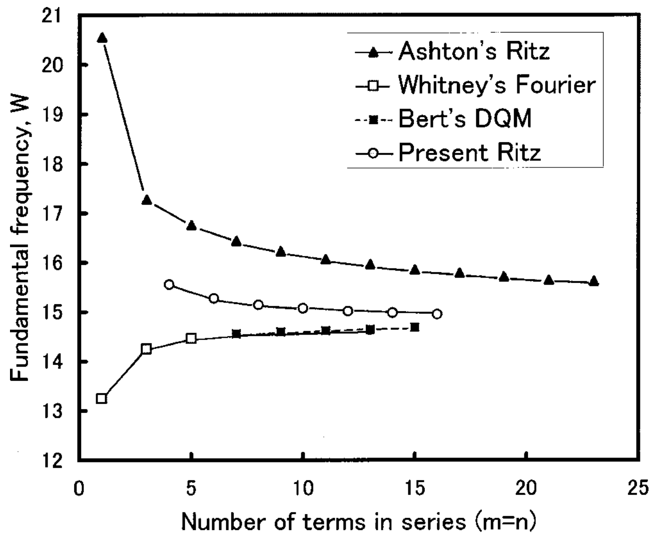

Fig. 1 Convergence of the fundamental frequency parameter $W=\Omega b^2(\rho/(Q_{22}h^2))^{1/2}$ by Ashton's Ritz method. Whiteney's Fourier, Bert's DQM, and the present Ritz method. (Data are replotted from Fig. 4 in [2].)

$$W=\omega b^2(\rho/Q_{22}h^2)^{1/2} \tag{2}$$

and are shown in Fig. 1 with those presented in Fig. 4 of [2]. It is observed that the present Ritz method gives much better upper bound than the Ritz result of Ashton and approaches closely to the lower bound of the Fourier analysis and DQ method. This figure therefore indicates that the convergence rate of the Ritz method is rather dependent on the choice of displacement functions.

The present author has an opinion that use of the Ritz method with (modified) polynomial functions yields very accurate upper bounds with advantages in applying to arbitrary boundary conditions and in computation time, when it is used with the following points in mind ([3]).

- The first few terms of the polynomial (say, ten) give rapid convergence of the solution, but the use of higher order polynomials (say, 20 or more terms) tends to make the eigenvalue equation numerically unstable, unless it is somehow modified.
- The plate region considered should have a regular plan, such as rectangular and elliptical plates. For plates of irregular geometry, e.g., with cutouts or L-shaped plates, the solution accuracy deteriorates.

In summary, the Ritz method with modified polynomials is a valuable and recommendable approach. The only problem is that because it is very easy to use and guarantees good accuracy, one cannot escape this easiness and does not create new methodology.

References

[1] Narita, Y., 2000, "Combinations for the Free-Vibration Behaviors of Anisotropic Rectangular Plates Under General Edge Conditions," ASME J. Appl. Mech., **67**, pp. 568–573.
[2] Bert, C. W., Wang, X., and Striz, A. G., "Convergence of the DQ Method in the Analysis of Anisotropic Plates," J. Sound Vib., **170**, pp. 140–144.
[3] Narita, Y., 1995, "Series and Ritz-Type Buckling Analysis," *Buckling and Postbuckling of Composite Plates*, G. J. Turvey and I. H. Marshall, eds., Chapman and Hall, London, p. 56.

