An Experimental Surface-Wave Method for Recording Force-Time Curves in Elastic Supports[1]

ENRICO VOLTERRA.[2] This paper is an important contribution in the field of stress-wave propagation. It furnishes a method for recording force-time curves when the well-known method, based on the use of the Hopkinson-Davies compression bar, fails because of dispersion effects connected with short pulses. In fact, this new method records Rayleigh surface waves propagated on the plane surface of a solid block, which waves are not affected by dispersion. For recording the short pulses produced by the impact of small hard-steel balls on the plane surface of the block, this method utilizes techniques already developed and used by one of the authors of this paper and described in previous papers [1, 2].[3] Another important feature of this paper is the experimental verification of Lamb's theory of propagation of surface waves on elastic solids [3].

Concerning the verification of Hertz's theory of impact, in addition to the works mentioned by the authors in their bibliography, two other papers could be mentioned. The first is by C. V. Raman [4] on transverse impact of elastic spheres on horizontally held infinitely extended plates of finite thickness. Raman derived a formula for the coefficient of restitution expressed as a function of the elastic constants and densities of the materials, the diameters of the spheres, the thicknesses of the plates, and the velocities of impact. In Raman's paper [4], experimental results of hard-steel balls dropped on glass plates were compared with the theoretical ones. The second paper is by C. Zener [5]. In Zener's paper a theory is developed for the study of short-duration impacts of spheres on large plates. Since the impacts were of short duration, the effects of the reflection from the boundaries of the plates were neglected.

Table 1 of this discussion shows the principal results of some experiments, carried on under the sponsorship of the Office of Naval Research, on the longitudinal elastic impact of steel balls on a very long cylindrical steel bar [6, 7]. Balls of diameters ranging from 1 to 2 in. were dropped from heights ranging from 7.5 to 30 cm to impact longitudinally a 1-in-diam bar. The stress-time curves produced during the impacts were obtained from an oscilloscope record of the pulse waves measured by strain gages fixed on the impacted bar. The experimental results are compared with the theoretical results obtained by applying Hertz's theory of impact. The nonlinear differential equation which was used in deriving the theoretical results to be compared with the experimental ones was the same as the one derived independently a year later by Goodier and Ripperger [6, 7]. The writer believes that the lack of agreement between theoretical and experimental results indicated in Fig. 6 of the paper is probably because of the fact that the heights of drops were much larger and the times of contact much shorter than those used in Table 1.

The writer wishes to congratulate the authors upon the work they have done in connection with this paper.

[1] By J. N. Goodier, W. E. Jahsman, and E. A. Ripperger, published in the March, 1959, issue of the JOURNAL OF APPLIED MECHANICS, vol. 26, TRANS. ASME, vol. 81, pp. 3–7.

[2] Professor, Department of Engineering Mechanics, The University of Texas, Austin, Tex. Mem. ASME.

[3] Numbers in brackets designate References at end of discussion.

References

1 E. A. Ripperger, "The Propagation of Pulses in Cylindrical Bars—An Experimental Study," Proceedings of the First Midwestern Conference on Solid Mechanics, Urbana, Ill., 1953, pp. 29–39.

2 E. A. Ripperger, "A Piezoelectric Strain Gage," *Proceedings of the Society for Experimental Stress Analysis*, vol. 12, 1954, pp. 117–124.

3 H. Lamb, "On the Propagaton of Tremors Over the Surface of an Elastic Solid," *Philosophical Transactions of The Royal Society of London*, series A, vol. 203, 1904, pp. 1–12.

4 C. V. Raman, "On Some Applications of Hertz's Theory of Impact," *Physical Review*, vol. 15, 1920, pp. 277–284.

5 C. Zener, "The Intrinsic Inelasticity of Large Plates," *Physical Review*, vol. 59, 1951, pp. 669–673.

6 R. A. Eubanks, D. Muster, and E. G. Volterra, "An Investigation on the Dynamic Properties of Plastics and Rubberlike Materials," Contract No. N7 ONR-064369 with Illinois Institute of Technology, Chicago, Ill., June 1952.

7 C. S. Barton, E. G. Volterra, and S. Citron, "On Elastic Impacts of Spheres on Long Rods," Proceedings of the Third U. S. Congress of Applied Mechanics, ASME, 1958, pp. 89–94.

Authors' Closure

Professor Volterra's discussion carries further the question of the applicability of the Hertz theory of impact of elastic spheres to impacts of spheres on other bodies—plane surfaces of large blocks, thin plates, and ends of cylindrical bars. In all these cases the conditions of the Hertz theory are not met for the reason pointed out in the paper in connection with the block, and yet in some a close correspondence of durations is found experimentally, and a more or less close correspondence of force magnitudes. In view of the violation of the Hertz conditions, the authors have taken the view that it is this *agreement* which calls for explanation,

Table 1 Comparison between experimental and theoretical results for impact tests

Ball diam, in.	Ball height, cm	Duration of pulse, microsec: Theoretical	Duration of pulse, microsec: Experimental	Duration of pulse, microsec: Difference	Maximum amplitude, megadynes/cm²: Theoretical	Maximum amplitude, megadynes/cm²: Experimental	Maximum amplitude, megadynes/cm²: Difference	Peak times, microsec: Theoretical	Peak times, microsec: Experimental	Peak times, microsec: Difference
1.00	30	75	78	+4.0	141	153	+8.5	35	37	+5.7
	22.5	77	79	+2.6	119	120	+0.1	36	37	+2.8
	15	80	82	+2.5	92	97	+5.4	42	40	−4.8
	7.5	86	86	0.0	62	59	−4.8	41	40	−2.4
1.25	30	95	97	+2.1	202	212	+5.0	43	46	+7.0
	22.5	98	102	+4.1	171	181	+5.9	42	46	+8.7
	15	102	105	+2.9	135	141	+4.5	46	47	+2.2
	7.5	108	110	+1.9	90	94	+4.4	50	51	2.0
1.50	30	120	124	+3.3	270	284	+5.2	51	54	+5.9
	22.5	121	123	+1.7	226	229	+1.2	52	54	+3.8
	15	126	129	+2.4	178	187	+5.1	55	59	+7.3
	7.5	134	136	+1.5	120	122	+1.7	59	59	0.0
1.75	30	141	142	+0.1	330	353	+6.1	58	61	+5.2
	22.5	146	147	+0.1	280	302	+7.9	59	63	+6.8
	15	151	151	0.0	223	256	+14.8	62	65	+4.8
	7.5	161	158	−1.9	150	164	+9.3	67	68	+1.5
2.00	30	170	168	−1.2	392	401	+2.3	65	68	+4.6
	22.5	174	172	−1.2	335	349	−4.2	67	69	+3.0
	15	180	180	0.0	267	290	+8.6	70	74	+5.7
	7.5	190	198	+4.2	181	183	+1.1	76	79	+3.9

rather than any lack of agreement. Hunter (Ref. [11] of the paper) has thrown some light on this for the block, and through his paper connection is made with the earlier work of Zener and Raman.

It may be, as Professor Volterra believes and as the experimental values he gives suggest, that experimental curves of the type shown in Fig. 6 of the paper would be in closer agreement with the Hertz curve if plastic flow did not occur. Yet the fact that the results actually obtained were closely repeatable in a large number of successive impacts on the same target seems to mean that plastic flow was not an important factor. A more extended investigation would be required to resolve these difficulties. The main purpose of the paper is to present the new method, with its present imperfections, in order that its potentialities as an adjunct to the Hopkinson pressure bar may be assessed. The authors thank Professor Volterra for his contribution to such an assessment.

A Mathematical Model Depicting the Stress-Strain Diagram and the Hysteresis Loop[1]

JoDEAN MORROW.[2] This paper is of great interest to the writer. About a year ago, in the course of a study of mechanical hysteresis as a criterion for fatigue failure, the writer expressed mathematically a curvilinear stress-strain curve as the integrated behavior of a set of "domains" each having different flow-stress properties. The subject was discussed subsequently at a graduate seminar on flow and fracture under the title, "Yield by Domains." The concepts involved are basically the same as presented by the author. The approach, however, is from the opposite direction; that is, a mathematical expression for a curvilinear stress-strain curve of a polydomain mass is manipulated to obtain the frequency-distribution curve of the domain strengths. The development follows:

Yield by Domains. The assumption is made that a body is composed of small discrete domains each having a stress-strain response like the author's Fig. 3. The elastic modulus, E, is assumed to be the same for each domain, but each has a different yield point. It is presumed that as load is applied to a polydomain mass, all domains experience the same strain in the direction of loading. As the weaker domains yield, the distribution of stress across the body is not uniform. The weaker domains continue to deform without increase in stress while the stronger domains still strain elastically. For any given average stress on a polydomain mass of unit area, there will be a certain fraction, k, of the cross section which is composed of fully plastic domains. Under these conditions the stiffness of the body or the tangent modulus, E_t, will be

$$E_t = E(1 - k) \tag{1}$$

The total strain in the direction of loading, ϵ, of a body may be expressed as the sum of the elastic strain, ϵ_e, and the plastic strain, ϵ_p:

$$\epsilon = \epsilon_e + \epsilon_p \tag{2}$$

The strains ϵ_e and ϵ_p may be written as functions of stress, σ, as follows:

$$\epsilon = \frac{\sigma}{E} + \epsilon_f \left(\frac{\sigma}{\sigma_f}\right)^{1/n} \tag{3}$$

where ϵ_f and σ_f are known corresponding values of plastic strain and stress and where n is the slope of the log σ − log ϵ_f curve.[3]

Differentiating Equation (3) with respect to σ, the reciprocal of the tangent modulus is obtained.

$$\frac{1}{E_T} = \frac{d\epsilon}{d\sigma} = \frac{1}{E} + \frac{\epsilon_f}{n\sigma_f}\left(\frac{\sigma}{\sigma_f}\right)^{\frac{1}{n}-1} \tag{4}$$

Solving for E_T, substituting in Equation (1), and solving explicitly for k, the following is obtained:

$$k = \frac{1}{\dfrac{\sigma_f n}{E\epsilon_f}\left(\dfrac{\sigma}{\sigma_f}\right)^{1-1/n} + 1} \tag{5}$$

Since k is defined as the fraction of domains which have yielded at a particular stress, Equation (5) represents the cumulative frequency-distribution curve of the domain strengths.

The frequency-distribution curve may be obtained by differentiating Equation (5) once with respect to σ:

$$\frac{dk}{d\sigma} = \frac{\dfrac{(1-n)}{E\epsilon_f}\left(\dfrac{\sigma}{\sigma_f}\right)^{-1/n}}{\left[\dfrac{\sigma_f n}{E\epsilon_f}\left(\dfrac{\sigma}{\sigma_f}\right)^{1-1/n} + 1\right]^2} \tag{6}$$

The types of cumulative frequency and frequency curve obtained for Equations (5) and (6) for two types of materials are shown in Fig. 1 of this discussion.

Work is continuing on this subject. Space does not permit details, but in closing the writer wishes to make a few comments concerning the promise of such an approach.

For many years the concept of domain behavior has been a

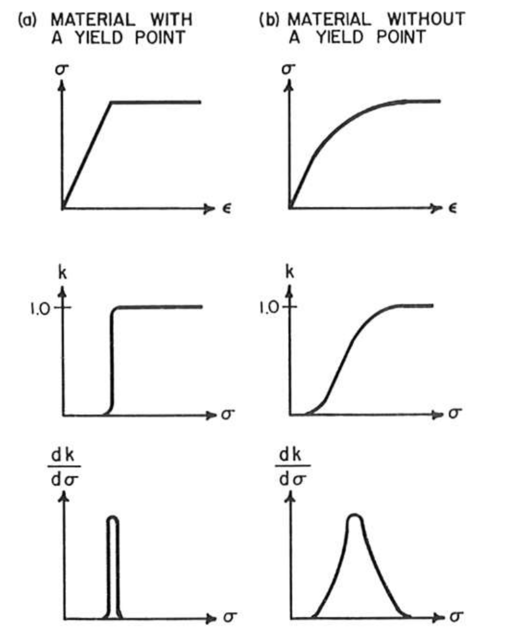

Fig. 1 Domain strength distribution for two types of materials

[1] By I. R. Whiteman, published in the March, 1959, issue of the JOURNAL OF APPLIED MECHANICS, vol. 26, TRANS. ASME, vol. 81, pp. 95–100.

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[3] The plastic-strain function, $\epsilon_f \left(\frac{\sigma}{\sigma_f}\right)^{1/n}$ is empirical. It presumes that a plot of log σ versus log ϵ_p is linear for the polydomain body.