



# Discussion

## Torsional Response of Rectangular Composite Laminates<sup>1</sup>

**Yi-Yuan Yu.<sup>2</sup>** It is gratifying to see the names of R. D. Mindlin and E. Reissner placed side by side to be given the credit jointly for the important contributions they both made to the theory of plates. Their contributions are of course closely related to each other. However, it is also important to recognize the differences between them. While Reissner (1945) deals with statics of plates, Mindlin (1951) is mainly concerned with dynamics and vibration of plates. Among other things, they treated the shear correction factor quite differently. By matching the frequency of the first antisymmetric mode of thickness-shear vibration given by his plate equations with that given by the exact elasticity theory, Mindlin (1951) arrived at a value of his shear factor equal to  $\pi^2/12$ , in contrast to the value of 5/6 given by Reissner. Mindlin later applied the same technique further to high frequency vibrations of crystal plates (1961). As pointed out recently by this writer (1989), Mindlin's later work (1961) on anisotropic plates has a close bearing upon

laminated composites, although it does not seem to have been cited as such.

The authors stated that both Reissner (1945) and Mindlin (1951) had solved the torsion problem, but a solution to the problem cannot be found in Mindlin's paper. The author's analysis is thus essentially an extension of Reissner's bending analysis to a laminated composite plate, and they arrived at the same value of 5/6 for the shear correction factor as Reissner's. Although the book on dynamic properties of composites by Read and Dean (1978) is further referenced in this paper, any dynamic torsional response of a plate has not really been covered.

### References

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<sup>1</sup>By C.-L. Tsai, I. M. Daniel, and G. Yaniv and published in the June 1990 issue of the *ASME JOURNAL OF APPLIED MECHANICS*, Vol. 57, pp. 383-387.

<sup>2</sup>Professor of Mechanical Engineering, Department of Mechanical and Industrial Engineering, New Jersey Institute of Technology, Newark, New Jersey 07102. Life Fellow ASME.