minimum value of frictional coefficient $\mu_0$ (dependent only on the strip Poisson's ratio). From [1, 2], $\mu_0$ is given as

$$\mu_0(\nu) = \frac{(\kappa + 1)\sin \pi\beta}{[(\kappa - 1)(\cos \pi\beta + 1) - 2(\kappa + 1)(\beta - 1) + 4(\beta - 1)^2]} \quad (1)$$

where $\beta$ is a solution of

$$2\kappa \cos \pi\beta - (\kappa^2 + 1) + 4(\beta - 1)^2 = 0; \qquad 0 < \beta < 1 \quad (2)$$

and

$$\kappa = \begin{cases} 3 - 4\nu, \text{ for plane strain, and} \\ (3 - \nu)/(1 + \nu), \text{ for plane stress.} \end{cases}$$

If $\mu \geqslant \mu_0(\nu)$, the exact solution of the problem corresponds to the case of $c = 1$ and should be identical to the solution presented in [2]. The solution presented by the authors is strictly applicable for the cases in which $\mu < \mu_0(\nu)$ and one obtains $c < 1$. Thus equation (57) in the paper is valid only for $\mu \leqslant \mu_0(\nu)$. Consequently, the correct representation of Fig. 6 must have a bounding curve corresponding to a plot of $\beta$ versus $\nu$, from equation (2) of this Discussion.

## References


1 Gupta, G. D., " An Integral Equation Approach to the Semi-Infinite Strip Problem," JOURNAL OF APPLIED MECHANICS, Vol. 40, TRANS. ASME, Vol. 95, Series E, 1973, p. 948.

2 Gupta, G. D., "The Problem of a Finite Strip Compressed Between Two Rough Rigid Stamps," JOURNAL OF APPLIED MECHANICS, Vol. 42, TRANS. ASME, Vol. 97, Series E, 1975, pp. 81–87.


# Authors' Closure

We would like to thank Dr. G. D. Gupta for his interest and the comments on the nature of stress singularity that arises in our investigation of the effect of friction on contact stresses in an elastic rectangle. Some comments are appropriate, however, for certain misunderstandings of our explanation of the results.

The statement quoted from the original paper by Dr. Gupta pertains to the "approximate solution" and *does not* refer to the main results obtained by the solutions of the simultaneous integral equations. This complete solution, indeed, confirms the points raised by Dr. Gupta and have been mentioned several places in the paper. This may also be clearly seen from Figs. 1 and 2 of the paper. The approximate solution is included in the paper for sake of comparison and is based upon the idea of R. D. Mindlin, which neglects the effect of friction on the contact pressure distribution in the slip zone. Mindlin's approximation has proved quite successful in several applications and is easy to apply. This approximation although, is quite good in the present case when considerable slip takes place, but is not so satisfactory when $c \to 1$. The part of discussion quoted by Dr. Gupta offers an explanation of this anomaly and should be read in conjunction with the preceeding statement. We are sorry that this was misunderstood.

Regarding the minimum values of the friction coefficient raised in the discussion, we are in complete agreement with Dr. Gupta. We had hoped that this was fairly apparent from a general study of the paper and did not include as a separate discussion for fear that the editorial office would wave the flag on exceeding the length requirements.

# On the Creep Rupture of a Tube and a Sphere[1]

**F. K. G. Odqvist.**[2] Using theory of Hayhurst and Leckie (*Journal of Mechanics and Physics of Solids*, 1973, Vol. 21, p. 431) author computes lower bounds on creep rupture time in cases of simple geometry and loading, both for "homogeneous" and "nonhomogeneous" damage rate law, material constant $\alpha$ in general being a fixed number between zero and one. Load is applied at time $t = 0$ and step-by-step calculations with respect to $t$ presented in dimensionless form. For a solid rod of circular cross section and a thick-walled tube, both in torsion with a constant torque, three cases were treated ($\alpha$ = 0, 0.5, 1.0) and in each case the ratio inner radius over outer radius of tube given the values 0 (solid rod), 0.5 (thick-walled tube), and 0.9 (thin-walled tube). A series of different values of material constants $n$ (Norton) and $\nu$ (Kachanov) were used. For example, the case $n = 4$, $\nu = 3$, $\alpha = 0$ for a solid rod, the dimensionless times to rupture were estimated to be 0.288 and 0.412 for a homogeneous and a nonhomogeneous maximum shear damage law, respectively. In this particular case, the ratio $\rho$ = estimated rupture time for nonhomogeneous damage law over that for homogeneous damage law turns out to be very much the same for the solid rod and the thick-walled tube for all values of $\alpha$ considered. In fact, from Table 2 may be inferred, for the solid rod: $\rho = 0.412/0.288 = 1.43$ for $\alpha = 0$. Similarly $\rho = 1.23$ for $\alpha = 0.5$ and $\rho = 0.98$ for $\alpha = 1$. In the case of the thick-walled tube was obtained, correspondingly: $\rho = 1.44$ for $\alpha = 0$, $\rho = 1.24$ for $\alpha = 0.5$, and $\rho = 0.99$ for $\alpha = 1$. Note that the figure 0.412 is taken from the table, whereas the text under "Conclusions" presumedly shows the erroneous figure 0.142:—thus the author's conclusion "that creep rupture time of a structural element could be significantly effected by the choice of damage law" hardly seems to be justified but, possibly, for the limiting case of the solid rod. Conversely, the dependence of creep rupture time on the material constant $\alpha$ seems to be quite insignificant. The difference between the time to rupture and time for the first crack to appear is so small that the failure occurs almost instantaneously, and this difference appears to be insensitive to the value of $\alpha$, in accordance with author's conclusion. Lower bounds on the rupture time for a hollow sphere creeping under constant internal pressure were also obtained, but in the case $\alpha = 0$ only.

# Author's Closure

The author would like to thank Professor Odqvist for his valuable comment.

# Parametric and Combination Resonances of a Pipe Conveying Pulsating Fluid[1]

**Takuzo Iwatsubo.**[2] The authors have reported some very interesting analytical work on the parametric and combination resonances of a pipe conveying pulsating fluid. The presence of combination resonances is open to question.

First, it is concluded in the left side of page 4 and the conclusions that the combination resonances appear to involve only the difference. But our results for the cantilevered beam are not the same as this result because from our results the combination resonances of the sum and the difference type appear as shown in Table 1.

From our analytical result, only the combination resonances of the difference type cannot occur.

[1] By R. P. Goel and published in the September, 1975, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 42, TRANS. ASME, Vol. 97, Series E, pp. 625–629.

[2] Professor, Torstensonsvägen 7D, S-18264, Djursholm, Sweden.

[1] By M. P. Paidoussis and C. Sundarajan and published in the December, 1975 issue of the JOURNAL OF APPLIED MECHANICS, Vol. 42, No. 4, TRANS. ASME, Vol. 97, Series E, pp. 780–784.

[2] The Faculty of Engineering, Kobe University, RokkoNada Kobe, Japan.



  

Table 1 Parametric resonance and combination resonance of sum and difference type ($\omega_i$: eigenfrequency)

| Mode no. | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | $2\omega_1$ | $\omega_2 - \omega_1$ | $\omega_1 + \omega_3$ | $\omega_4 - \omega_1$ | $\omega_1 + \omega_5$ |
| 2 | | $2\omega_2$ | $\omega_3 - \omega_2$ | $\omega_2 + \omega_4$ | $\omega_5 - \omega_2$ |
| 3 | | | $2\omega_3$ | $\omega_4 - \omega_3$ | $\omega_3 + \omega_5$ |
| 4 | | | | $2\omega_4$ | $\omega_5 - \omega_4$ |
| 5 | | symmetry | | | $2\omega_5$ |

Second, it is concluded that for the cantilevered pipes the parametric resonances are selectively associated with only some of the modes of the system. Is it the general conclusion for a pipe conveying pulsating fluid? If it is, I would like to know the reason.

## Authors' Closure

We are very grateful to Professor Iwatsubo for his discussion of our paper.

Concerning his first point, there appears to be some contradiction between the second and third paragraphs of the Discussion. However, we agree with the first statement made by the discusser that, for cantilevered columns, both sum and difference-type combination resonances are possible. In our paper, concerning cantilevered pipes conveying fluid, we were careful to say that "the combination resonances *appear* to involve the differences, rather than the sums." We have not made a special study of this, and our supposition was based on the consideration that for the low frequencies involved it is more likely that the combination resonances be of the difference rather than the sum type, since in the latter case $(\omega_i + \omega_j)/k$ near zero would imply very large values of $k$.

Concerning the second point, the limited extent of our calculations does not allow us to say with certainty that, let us say, first-mode parametric resonances are impossible for *all* possible sets of system parameters. However, we have never found such instabilities in our analysis, nor has it ever been found in the experiments [1].[3] A possible explanation is this. In the cantilevered pipe the Coriolis acceleration acts effectively as a damping force and the effective damping varies from one mode to another [2]. It is certainly possible that some modes are simply too heavily "damped" by the Coriolis effect to exhibit parametric resonances, either over a wide range of flow velocities or for all flow velocities.

### References

1 Paidousis, M. P., and Issid, N. T. "Experiments on Parametric Resonance of Pipes Containing Pulsatile Flow," JOURNAL OF APPLIED MECHANICS Vol. 43, No. 2, TRANS. ASME, Vol. 98, Series E, pp. 198–202.

2 Paidoussis, M. P., and Issid, N. T. "Dynamic Stability of Pipes Conveying Fluid," *Journal of Sound and Vibration*, Vol. 33, 1974, pp. 267–294.

[3] Numbers in brackets designate References at end of Closure.

# Measurement of Angular Acceleration of a Rigid Body Using Linear Accelerometers[1]

**Y. King Liu.**[2] The authors are to be commended for posing a most interesting inverse problem in rigid body mechanics, i.e., is it possible to infer the total acceleration of a rigid body from strategically placed linear accelerometers? The writer wishes to indicate however that the equations (3)–(5) of the authors' paper in fact can be shown analytically to be unstable. Equations (3)–(5) can be rewritten as

$$\dot{\omega}_x = \alpha_x - \omega_y\omega_z \qquad (a)$$

$$\dot{\omega}_y = -\alpha_y + \omega_x\omega_z \qquad (b)$$

$$\dot{\omega}_z = \alpha_z - \omega_x\omega_y \qquad (c)$$

where $\alpha_x = (A_{z1} - A_{z0})/\rho_{y1}$, $\alpha_y = (A_{z2} - A_{z0})/\rho_{x2}$ and $\alpha_z = (A_{y2} - A_{y0})/\rho_{x2}$. The stability analysis of the system of equations just given can be obtained through the calssical Routh-Hurwitz criterion [1].[3] A state of equilibrium may be represented by a singular point, i.e., where all the derivatives of the dependent variable with respect to time are simultaneously zero, i.e.,

$$\alpha_x - \omega_y^0\omega_z^0 = 0 \qquad (d)$$

$$-\alpha_y - \omega_x^0\omega_z^0 = 0 \qquad (e)$$

$$\alpha_z - \omega_x^0\omega_y^0 = 0 \qquad (f)$$

where $\omega_x^0$ $\omega_y^0$ and $\omega_z^0$ denote a set of equilibrium values for the dependent variables. Consider small perturbations, $\xi_i$, defined by the following equation:

$$\omega_x = \omega_x^0 + \xi_x \qquad (g)$$

$$\omega_y = \omega_y^0 + \xi_y \qquad (h)$$

$$\omega_z = \omega_z^0 + \xi_z \qquad (i)$$

Substituting the foregoing into (*a*)–(*c*) yields the following:

$$\dot{\xi}_x = \omega_y^0\xi_x - \omega_z^0\xi_y \qquad (j)$$

$$\dot{\xi}_y = \omega_x^0\xi_z + \omega_z^0\xi_x \qquad (k)$$

$$\dot{\xi}_z = -\omega_x^0\xi_y - \omega_y^0\xi_x \qquad (l)$$

It has been shown by Liapunov [2] that if the real parts of the roots of the characteristic equation of the foregoing system are negative then the corresponding equilibrium state is stable or if at least one root has a positive real part the equilibrium is unstable. The characteristic equations may now be written as follows:

$$\begin{vmatrix} -\lambda & -\omega_z^0 & -\omega_y^0 \\ \omega_z^0 & -\lambda & \omega_x^0 \\ -\omega_y^0 & -\omega_x^0 & -\lambda \end{vmatrix} = 0$$

or

$$-\lambda^3 + (-\omega_x^{02} + \omega_y^{02} + \omega_z^{02})\lambda + 2\omega_x^0\omega_y^0\omega_z^0 = 0 \qquad (m)$$

The Routh-Hurwitz criterion requires that all coefficients in the characteristic equation exist and are positive. In (*m*), the $\lambda^2$ term is missing. Thus the given set of differential equations is *unstable*.

Since the solution to the foregoing set of equations is unique given the initial values, in spite of its being unstable, one can further assert that no amount of additional manipulation or clever placement of the six accelerometers would yield a set of equations which will be stable.

By going to the nine accelerometer scheme the authors have in fact added 3 pieces of information, i.e., equations (6)–(8) to the original (3)–(5). The authors' equations (9)–(11) now contain 9 temporal measurements and straightforward algebra to obtain the desired result. The major disadvantage is obviously the addition of 3 channels of data acquisition and analysis.

[1] By A. J. Padgaonkar, K. W. Krieger, and A. I. King, and published in the September, 1975, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 42, TRANS. ASME, Vol. 97, Series E, pp. 552–556.

[2] Professor and Director, Biomechanics Laboratory, Tulane University Medical School, New Orleans, La.

[3] Numbers in brackets designate References at end of Discussion.