

DISCUSSION

and Bhatia and Nachbar [3]. All three papers deal with finite deformations in centrally loaded circular membranes; the first treats a linear elastic membrane loaded through a central circular plug, the second considers the load as a concentrated central force, and the last analyzes an elastic-plastic membrane loaded through a hemispherical indenter.

For moderate rotations of the membrane, the solutions given in the three papers just cited and the authors' paper should show close agreement for the traction-free region of the membrane. Have the authors carried out any comparisons between their results and those given in references [1-3]?

References

- 1 Schwerin, E., "Über Spannungen und Formänderung kreisringförmiger Membranen," *Z. Tech. Phys.*, No. 12, 1929, pp. 651-659.
- 2 Jahsman, W. E., Field, F. A., and Holmes, A. M. C., "Finite Deformations in a Prestressed, Centrally Loaded, Circular Elastic Membrane," *Proceedings, 4th U. S. National Congress of Applied Mechanics*, ASME, 1962, pp. 585-594.
- 3 Bhatia, N. M., and Nachbar, W., "Finite Indentation of Elastic, Perfectly Plastic Membranes by a Spherical Indenter," *AIAA Journal*, Vol. 6, No. 6, June 1968, pp. 1050-1057.

Authors' Closure

We appreciate the three papers which the discussor has brought to our attention. Upon reviewing these papers, we found that they all deal with Hookean materials, with one paper carrying the deformation beyond the elastic limit into the ideal perfectly plastic range, and assume small strain as well as moderate rotation.

Our paper is written on an entirely different basis both in philosophy and application. First of all, we deal with a material which is capable of finite deformation in its elastic range. The use of Mooney strain-energy function is intended to model a rubber membrane which cannot be described by Hooke's law even in the small strain range. Second, we are not interested in an approximate closed-form solution to this problem since it sometimes overstretchers limitations such as small strain, moderate rotation, etc. These assumptions do not seem acceptable in the indentation problem we consider. Our main interest lies in good methods and accurate numerical solutions to the "exact" equations.

Even in the small strain and moderate rotation range, no meaningful comparison, as suggested by the discussor, can be made unless we use Hooke's law which we do not believe to be pertinent to our problem.

Transient Interaction of Spherical Acoustic Waves and a Spherical Elastic Shell¹

B. S. BERGER.² The authors' paper, which gives some valuable insight into a technically interesting problem, is similar to an earlier paper by Mann-Nachbar [1],³ in that solutions are developed in terms of an integral transform in time and a series expansion of Legendre and Bessel functions in space. As noted in [1] the series representing the pressure of the fluid and the radial acceleration of the shell do not, in general, converge uniformly. It follows that for certain values of the variables θ and τ the inversion of the series does not converge to the solution. This difficulty, noted by Mann-Nachbar, applies to the present paper.

Considering that the sphere is rapidly loaded by the acoustic wave, a model of the shell based on the hollow sphere solution of linear elasticity might give a more accurate prediction of strain [2]. It appears that the authors' solution could be extended to the more exact case.

References

- 1 Mann-Nachbar, P., "The Interaction of an Acoustic Wave and an Elastic Spherical Shell," *Quarterly Journal of Applied Mathematics*, Vol. 15, Apr. 1957, pp. 83-89.
- 2 Berger, B. S., "Vibration of A Hollow Sphere in an Acoustic Medium," *JOURNAL OF APPLIED MECHANICS*, Vol. 36, No. 2, TRANS. ASME, Vol. 91, Series E, June 1969, pp. 330-333.

Authors' Closure

The series representing the pressure and acceleration do not converge uniformly near the incident, the reflected, and the refracted wave fronts due to Gibb's phenomenon. The complete solution of the title problem could be constructed by combining the series solution and some type of wave-front analysis, for instance, by applying the Watson's transform to the Laplace transform of the series solution and evaluating the inverse Laplace transform asymptotically for small values of time. The solution can be easily extended to the case involving a hollow elastic sphere with some additional computation effort.

¹ By H. Huang, Y. P. Lu, and Y. F. Wang, published in the March, 1971, issue of the *JOURNAL OF APPLIED MECHANICS*, Vol. 93, Series E, pp. 71-74.

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³ Numbers in brackets designate References at end of Discussion.