

The Scattering of Shock Waves by Cylindrical Cavities in Liquids and Solids¹

THOMAS L. GEERS.² In this paper, Dr. Harper has confirmed what some previous investigators have at least strongly suspected, viz., that a cavity in a fluid which is engulfed by a shock wave produces dramatic near-field pressure reduction in the fluid. Two papers by Baron,^{3,4} for example, deal with transient scattering by obstacles which present boundaries to the fluid that are first soft but then stiffen rapidly with decreasing obstacle volume. Each of these obstacles, therefore, constitutes a "cavity with a cushion." Unfortunately, Baron's computations pertain to a rather abruptly acting cushion, so that he finds, instead of pressure attenuation, pressure amplification (in terms of peak pressure).

At about the same time Dr. Harper was pursuing the studies reported in his paper, I was investigating the related problem of the scattering of acoustic pulses by a cylindrical shell.⁵ Shortly after performing the computations mentioned in his paper, I did a rather cursory study of Baron's problem, except that I used a gradually acting cushion. The results suggested that significant pressure reduction is possible when the incident wave pulse duration does not greatly exceed the time for the wave front to envelop the cavity. Furthermore, I found that a cushion is absolutely necessary to prevent, even for pulses of short duration, the ultimate collapse of the cavity and the associated generation of a large-amplitude bubble pulse.

With all phenomena considered, therefore, it seems that, while neither the abruptly acting cushion nor the infinitely soft cushion produces satisfactory pressure reduction in the fluid, the gradually acting cushion works effectively. I plan to report in greater detail on these matters in the near future.

N. R. ZITRON.⁶ I would like to begin my discussion with some comments on Dr. Harper's previous paper [1]⁷ which contains a detailed development of the theory. I regard the theoretical development as an important contribution to the literature because it shows how the method of matched asymptotic expansions which has proven useful in fluid flow can be extended to diffraction of waves. Chen [2] had obtained a result previously which exhibited a logarithmic time decay. Harper's solution showed that Chen's result was the inner expansion. Thus the region exterior to the cylinder is decomposed into two distinct regions with different physical significance. The matching of the outer and inner expansions was accomplished by means of van Dyke's rule [3] which states that the inner expansion (to order α) of the outer expansion (to order β) is equivalent to the outer expansion (to order β) of the inner expansion (to order α). The analysis is carried out without reference to the exact solution and is, therefore, not restricted to circular cylinders. Thus the treatment of other shapes is facilitated.

I would like to point out an important application of the present investigation. It is possible to protect materials from spallation damage by drilling cavities in them. This suggests a paradox; namely, that solid bodies, which are weakened against static loads by removing part of the material, are strengthened against elastic

waves by this removal. A related investigation was carried out by Mok [4] who showed that a grating of fibers in a material causes elastic waves to attenuate.

I would like to suggest the possibility of additional experiments for the case of elliptic cylinder whose theory was developed in Harper's previous investigation.

In closing, I would like to compliment Dr. Harper on the thoroughness of his investigation.

References

- 1 Harper, E. Y., *Journal of Mathematics and Physics*, Vol. 10, 1969, p. 1799.
- 2 Chen, Y. M., *International Journal of Engineering Sciences*, Vol. 2, 1964, p. 417.
- 3 van Dyke, M. D., *Perturbation Methods in Fluid Mechanics*, Academic Press, Inc., New York, 1964.
- 4 Mok, C. H., *Journal of the Acoustical Society of America*, Vol. 46, 1969, p. 631.

Response of an Elastic Half Space to Expanding Surface Loads¹

K. I. BEITIN.² The author states in his introduction that, contrary to the discussor's work [1],³ he has found that the ring load expanding steadily at the Rayleigh wave speed does not cause a singular response of the entire half space. This statement is grossly inaccurate. The discussor's work had dealt with a point load moving at variable velocity and had shown no singular behavior of the entire half space. The author's remark may have been made in reference to an introductory statement in the discussor's work which should have been qualified. Unfortunately, the statement had implied that any surface load moving steadily at the Rayleigh wave speed produces a singular response of the entire half space. The introductory statement was made to point out the usual difficulties encountered in the steady-state solutions of line loads moving at the Rayleigh wave speed. The author had previously taken issue with this statement [2] and had pointed out that a transient solution to the foregoing problem does not exhibit a singular behavior over the entire region. If this is what the author is referring to when he speaks of contradiction between the two works then his choice of words is obviously misleading.

References

- 1 Beitin, K. I., "Response of an Elastic Half Space to a Decelerating Surface Point Load," *JOURNAL OF APPLIED MECHANICS*, Vol. 36, TRANS. ASME, Vol. 91, Series E, Dec. 1969, pp. 819-826.
- 2 Gakenheimer, D. C., Discussion in *JOURNAL OF APPLIED MECHANICS*, Vol. 37, TRANS. ASME, Vol. 92, Series E, Sept. 1970, pp. 876-877.

Author's Closure

The author believes that his reference to the discussor's paper (reference [1] of the Discussion) is perfectly accurate. The Introduction of the discussor's paper includes an unqualified, and in the context of his paper, incorrect statement about loads moving at the Rayleigh wave speed. Since this statement is one of the premises for the discussor's work, it is particularly unfortunate that it is not properly qualified. The author discussed the statement previously (reference [2] of the Discussion), and then revived the issue in his paper because he had another example, which included numerical results, to support his con-

¹ By E. Y. Harper, published in the March, 1971, issue of the *JOURNAL OF APPLIED MECHANICS*, Vol. 93, Series E, pp. 190-196.

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³ Baron, M. L., "Response of Nonlinearly Supported Boundaries to Shock Waves," *JOURNAL OF APPLIED MECHANICS*, Vol. 24, No. 4, TRANS. ASME, Vol. 79, Series E, Dec. 1957, pp. 501-505.

⁴ Baron, M. L., "Response of Nonlinearly Supported Cylindrical Boundaries to Shock Waves," *JOURNAL OF APPLIED MECHANICS*, Vol. 28, No. 1, TRANS. ASME, Vol. 83, Series E, Mar. 1961, p. 135.

⁵ Geers, T. L., "Excitation of an Elastic Cylindrical Shell by a Transient Acoustic Wave," *JOURNAL OF APPLIED MECHANICS*, Vol. 36, No. 3, TRANS. ASME, Vol. 91, Series E, Sept. 1969, pp. 459-461.

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⁷ Numbers in brackets designate References at end of Discussion.

clusion. Although the author considers expanding loads shaped like a ring and a disk and the discussor a traveling point load, the conclusion is still valid.

The author apologizes to the discussor if he is offended by the reference to his work. However, the author would like to emphasize the need for accurate statements in a paper, even in the Introduction. Inaccurate statements have a way of propagating for years throughout the literature.

On the Almost-Sure Stability of Linear Stochastic Systems¹

T. K. CAUGHEY² and A. H. GRAY, JR.³ Due to an error in both of the examples considered in the Brief Note the author appeared to demonstrate an improvement over the results of Infante.⁴ In both cases the error arose in describing the maximum eigenvalue of the matrix $[(F'P + PF)Q^{-1}]$,

$$\lambda_{\max} = \lambda_{\max}\{[(F'P + PF)Q^{-1}]\},$$

where the result given was only correct for positive $f(t)$.

In the first example utilized, the eigenvalues of $[(F'P + PF)Q^{-1}]$ can be expressed as

$$f(t)[(\pm\sqrt{\xi^2 + 1} - \xi)/2\xi].$$

When $f(t)$ is positive, the maximum eigenvalue is clearly found by using the plus sign in front of the radical; however, when $f(t)$ is negative, the minus sign must be used in front of the radical. As a result, the maximum eigenvalue can be expressed as

$$\lambda_{\max} = |f(t)| \frac{\sqrt{\xi^2 + 1}}{2\xi} - f(t)/2.$$

Taking the expected value of this expression, and utilizing the fact that $E[f(t)]$ is zero, results in the fact that equation (24) of the Brief Note should read

$$E[|f(t)|] < 2\xi/\sqrt{\xi^2 + 1},$$

which is not nearly as good as the result of Infante.⁴

In example two of the Brief Note, a similar error was made, and equation (26) of that Note should read

$$E[|g(t)|] < 2\xi/\sqrt{\xi^2 + 1},$$

which again is not comparable with the results of Infante.⁴ This latter correction, on example two, eliminates the problem discussed by the author about the apparent opposition to physical intuition of the result he obtained.

¹ By F. T. Man, published in June, 1970, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 37, Series E, No. 2, pp. 541-543.

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³ Associate Professor, Department of Electrical Engineering, University of California, Santa Barbara, Calif.

⁴ E. F. Infante, "On the Stability of Some Linear Nonautonomous Random Systems," JOURNAL OF APPLIED MECHANICS, Vol. 35, No. 1, TRANS. ASME, Vol. 90, Series E, Mar., 1968, pp. 7-12.

The Turbulence Characteristics of Two-Dimensional Wall-Jet and Wall-Wake Flows¹

F. F. Erian.² The authors have presented data very similar to what was reported in reference [6]. Few additions included the

¹ By S. C. Kacker and J. H. Whitelaw, published in the March, 1971, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 93, Series E, pp. 239-252.

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approximate measurement of u -spectra and the dissipation term in the turbulent energy equation. Also, the experiment was repeated for a larger jet lip thickness.

In the energy balance at $x/y = 10$, $t/y_c = 1.14$, and $\bar{U}_c/U_a = 2.3$ it is interesting to observe the behavior of the production and diffusion of turbulent energy. When the production term represented a gain of turbulent energy, the diffusion, not the dissipation, term represented the major loss term. On the other hand, in the region between the two different y -locations of the zero turbulent shear stress and the zero mean velocity gradient, the production was a loss term and the turbulent kinetic energy was maintained by diffusion. This behavior seems to be characteristic of that region where the viscous shear stress and the turbulent Reynolds stress have opposing effects.

The reported large scatter in the length scale profiles and the lack of similarity in their behavior make the prediction procedure of doubtful value.

H. FERNHOLZ.³ The paper provides a valuable approach to the experimental investigation of turbulent wall-wake and wall-jet flows. It is one of the few papers where mean and fluctuating quantities of the flow were measured extensively with the ratios of slot to free-stream velocity (U_c/U_a) and the slot to lip thickness (y_c/t) as parameters.

One of the most interesting but also most difficult regions of the flow is that close to the slot, especially when the lip thickness is of the order of the slot height. The authors did measure fluctuating quantities in this partly separated region and state quite correctly that measurements in this region should be regarded as qualitative if the turbulence intensity is in excess of about 20 percent. Since the process of turbulent mixing just downstream of the slot seems to be very important for practical applications it would be of considerable interest to correct the data by using the correction curve of Champagne [1]⁴ and for higher turbulence intensities that of Vagt [2] which was published in the meantime. It is thus possible to achieve more reliable data up to turbulence intensities of 30 percent. In this context it may be worthwhile to know the spectrum of turbulence intensities in the outer flow.

In order to provide further evidence in the case "Coincidence of position of zero shear stress and maximum velocity"—yes versus no—it would have been most instructive if the location of the values in question had been plotted and, if possible, somehow correlated.

Even if Bradshaw and Gee [3] had settled this issue I would not dare to extrapolate their measurements to flow configurations dominated by other parameter combinations. Once this problem is solved existing eddy viscosity models for simple wall jets can be tested and hopefully carried over to these rather complicated flow species.

References

- 1 Champagne, F. H., "Turbulence Measurements With Inclined Hot Wires," BSRL Flight Sciences Report 103, 1965.
- 2 Vagt, J. D., "Hot-Wire Measurement Technique in a Highly Turbulent Flow and the Calculation of Intensities," International Seminar for Heat and Mass Transfer, Herceg Novi, Yugoslavia, 1969.
- 3 Bradshaw, P., and Gee, M. T., "Turbulent Wall Jets With and Without an External Stream," R&M No. 3252, 1960.

B. G. NEWMAN.⁵ Kacker and Whitelaw present some new and useful turbulence measurements in wall jets. The accuracy of the hot-wire readings, particularly those which are difficult to measure, e. g., \bar{v}^2 and \bar{w}^2 , appears to be unusually good. High

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⁴ Numbers in brackets designate References at end of Discussion.

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