

## A Continuum Theory of Fluid Saturated Porous Media<sup>1</sup>

RAY M. BOWEN.<sup>2</sup> This paper concerns the thermodynamics of a mixture of a solid and several fluids. The authors adopt the field equations originally proposed by Truesdell [1]<sup>3</sup> and an entropy inequality for each constituent originally proposed by Eringen and Ingram [2, 3]. As described in the authors' Introduction, the results of Eringen and Ingram have been shown to be unacceptable by Müller [4]. Müller [4, section 11] pointed out that Eringen and Ingram did not obtain all of the thermodynamic results from their inequality. As Müller indicated, these additional results show that Eringen and Ingram's formulation forces certain very special formulas to hold. One of Müller's main conclusions was that it is essential in a theory of diffusion to allow for a dependence on strain gradients in the constitutive equations. In a certain sense, the paper under discussion seeks to avoid the special results of Eringen and Ingram by adopting Müller's suggestion.

The real question with the Eringen and Ingram formulation is whether or not an entropy inequality for each constituent is the correct form of the second law of thermodynamics for mixtures. It is felt by some authors that an entropy inequality should be stated only for the mixture. (See, for example, Müller [4], Truesdell [5], Bowen and Wiese [6], and Green and Naghdi [7].) While it is difficult to assert absolutely that a particular entropy inequality is correct, it is often not difficult to conclude that a particular statement is incorrect. Often an incorrect entropy inequality will produce results which prohibit the formulation from being consistent with generally accepted classical results. This observation seems to apply to the results of Bedford and Ingram. In a theory where viscous effects are felt to be unimportant, it is natural to omit velocity gradients from the constitutive equations. In this case the authors' equation (37) yields, among other things,

$$\frac{\partial \hat{\psi}_{(\xi)}}{\partial \rho_{(\lambda)}} = 0 \quad (1)$$

for  $\xi \neq \lambda$ , and

$$\frac{\partial \hat{\psi}_{(\xi)}}{\partial x_{(s)}^m} = 0 \quad (2)$$

for  $\xi \neq s$ . According to (1), (2), and the authors' (33), for a mixture consisting of a single elastic solid and a finite number of nonviscous fluids it is necessary for

$$\psi_{(s)} = \hat{\psi}_{(s)}(x_{(s)}^m, \theta_{(s)}) \quad (3)$$

for the solid and

$$\psi_{(\xi)} = \hat{\psi}_{(\xi)}(\rho_{(\xi)}, \theta_{(\xi)}) \quad (4)$$

for the fluids. As pointed out by Müller [4, section 8] and Bowen and Wiese [6, section 5], a mixture theory which forces

(3) and (4) to hold cannot be accepted. Equation (4) leads, for example, to a chemical potential for each fluid which depends only on the density of that fluid. This special type of dependence is inconsistent with classical thermochemistry.

Since they considered a theory with viscous effects, Bedford and Ingram avoided the unacceptable results (3) and (4). However, it does seem undesirable that viscous effects are essential to their formulation. Perhaps this result indicates their entropy inequality is not correct. Both Müller [4] and Bowen and Wiese [6] have presented thermodynamic formulations which postulate an entropy inequality for the mixture and which do not produce the results (3) and (4) when viscous effects are omitted.

### References

- 1 Truesdell, C., "Sulle Basi Della Thermomeccanica," *Rend. Lincei*, Vol. 22, No. 8, 1957, p. 33.
- 2 Eringen, A. C., and Ingram, J. D., "A Continuum Theory of Chemically Reacting Media—I," *International Journal of Engineering Science*, Vol. 3, 1965, p. 197.
- 3 Ingram, J. D., and Eringen, A. C., "A Continuum Theory of Chemically Reacting Media—II. Constitutive Equations of Reacting Fluid Mixtures," *International Journal of Engineering Science*, Vol. 5, 1967, p. 289.
- 4 Müller, I., "A Thermodynamic Theory of Mixtures of Fluids," *Archive for Rational Mechanics and Analysis*, Vol. 26, 1967, p. 118.
- 5 Truesdell, C., *Rational Thermodynamics*, McGraw-Hill, New York, 1969.
- 6 Bowen, R. M., and Wiese, J. C., "Diffusion in Mixtures of Elastic Materials," *International Journal of Engineering Science*, Vol. 7, 1969, p. 689.
- 7 Green, A. E., and Naghdi, P. M., "On Basic Equations of Mixtures," *Quarterly Journal of Mechanics and Applied Mathematics*, Vol. 22, 1969, p. 427.

### Authors' Closure

The authors wish to thank Professor Bowen for his comments. His fine and extensive work in the continuum theory of mixtures requires that his remarks be given particularly careful consideration.

Professor Bowen's observation that neglect of the constituent velocity gradients causes the Eringen and Ingram entropy inequality to impose restrictions on the constitutive functions which are at variance with classical results is correct. This fact permits two interpretations. The first, which Professor Bowen suggests, is that the entropy inequality is incorrect. An alternative interpretation is that the restrictions imposed by the Eringen-Ingram inequality imply that velocity gradients are essential constitutive variables in a theory of diffusing materials.

On intuitive grounds, it is difficult to conceive of a diffusive process in which viscous dissipation would not play a role. For example, in diffusion of a fluid through a solid, the diffusive force exerted on the solid by the fluid depends entirely on the viscous nature of the fluid. If the fluid was perfect, there would be no diffusive force. We suggest that there are no diffusing materials where "... viscous effects are felt to be unimportant."

In the light of this interpretation, we do not find surprising the fact that the dissipation inequality predicts unacceptable results if essential dissipational variables are ignored. In addition, we feel that theories of mixtures developed in the past in which diffusional interactions depending on relative velocities have been included but velocity gradients have been neglected are inconsistent in that regard.

<sup>1</sup> By A. Bedford, and J. D. Ingram, published in the March, 1971, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 93, Series E, pp. 1-7.

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<sup>3</sup> Numbers in brackets designate References at end of Discussion.