the case of constant tension. The case of the cable with non-negligible weight has a rather extensive bibliography for both the static [1, 2, 3][3] and the dynamic cases [4, 5, 6]. Eringen [7] shows that extensional deformations need not be neglected in deriving the author's equation (17). In reference [3], the methods developed by Moser [8] and Wasow [9] for singular perturbation differential equations have been used to obtain solutions of this equation similar to those of equations (41) and (42) of the author's paper but for the more general case of linearly varying axial tension caused by the weight of the cable. It is also shown there that the author's restriction to small angles is not necessary and that the effects of constant horizontal and vertical forces can be absorbed into the boundary conditions. The results he obtains, indicating that superposition can be used for calculating bending moments in the presence of lateral forces even for rather large angles, should be of interest for the important practical problem of offshore casings loaded laterally by tidal currents.

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R. SCHMIDT[4] and D. A. DaDEPPO.[5] The nonlinear differential equations governing the three problems considered by the author can also be linearized with the aid of Chebyshev polynomials [10–12].[6] Such linearization yields more accurate results than the so-called "small-angle approximation," i.e., linearization with the aid of Maclaurin expansions.

Let us imagine the cable extended below the horizontal plane of fixity and take

$$\varphi = \gamma - \theta, \qquad -\gamma \leq \varphi \leq \gamma$$

The author's equation (18) then becomes

$$\varphi'' - \lambda^2 \sin \varphi = 0 \tag{1}$$

where primes denote derivatives with respect to s. According to [13] of this Discussion,

$$\sin \varphi \approx \frac{2}{\gamma} J_1(\gamma)\varphi, \qquad \cos \varphi \approx J_0(\gamma) \tag{2}$$

where J_0 and J_1 are the ordinary Bessel functions of the zeroth and first order, respectively. Substituting (2) into (1) and letting

$$k^2 = \frac{2}{\gamma} \lambda^2 J_1(\gamma) = \frac{2T^*}{\gamma EI} J_1(\gamma) \tag{3}$$

we obtain the approximate equation

$$\varphi'' - k^2\varphi = 0 \tag{4}$$

which is satisfied by

$$\varphi = C_1 \sinh (ks) + C_2 \cosh (ks) \tag{5}$$

From the conditions $\varphi = \gamma$ at $s = 0$; and $\varphi = 0$, $M = -EI\varphi' = 0$ at $s = L$,

$$C_1 = -\gamma \coth (kL) = -\gamma \tanh (kL), \qquad C_2 = \gamma \tag{6}$$

Hence,

$$\tanh (kL) = 1 \tag{7}$$

$$\varphi = \gamma[\cosh (ks) - \sinh (ks)] \tag{8}$$

$$M_0 = [2\gamma J_1(\gamma) EIT^*]^{1/2} \tag{9}$$

For example, for $\gamma = 2$ ($\gamma \approx 114.59$ deg), exact $M_0 = 1.683(EIT^*)^{1/2}$; from equation (9), $M_0 = 1.519(EIT^*)^{1/2}$; and by the small-angle approximation, $M_0 = 2(EIT^*)^{1/2}$. Of course, the accuracy of the method is much better for smaller angles.

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Author's Closure

The author would like to thank Professors Plunkett, Schmidt, and DaDeppo for their informative comments and the lists of additional references they have provided which include several related papers of which the author was unaware. In particular, it is gratifying to the author to know that some of the results of the paper have a wider range of application than indicated in the paper.

Flow in a Two-Dimensional Channel With a Rectangular Cavity[1]

ZEËV ROTEM.[2] The authors are to be complimented on their investigation of this important problem which has a direct bearing

[3] Numbers in brackets designate References at end of Discussion.

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[5] Professor of Civil Engineering, The University of Arizona, Tucson, Ariz.

[6] Numbers in brackets designate Additional References at end of Discussion.

[1] By U. B. Mehta and Zalman Lavan, published in the December, 1969, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 36, TRANS. ASME, Vol. 91, Series E, pp. 897–901.

[2] Professor, Department of Mechanical Engineering, University of British Columbia, Vancouver, B. C., Canada.

upon static hole errors for low and moderate Reynolds numbers Re.

It is interesting to compare the authors' results with those of previous investigators who examined more or less the same problem. Thus Thom and Apelt [1][3] assumed a Poiseuille-type flow over their cavity of aspect ratio 2:1, and Re = 5. While these early results may not be particularly accurate, they are at variance with the results of the authors, and some discussion of this would seem valuable. A particular point in case would perhaps be the surprising degree of symmetry retained at Re = 100 in the authors' results and the shape of the separating streamline. Burggraf [2] apparently does not find such a degree of symmetry in his case (which, however, assumed a different type of shear flow).

An interesting comparison could also be made with the various models which impose a rate of shear at the cavity upper ("free") surface [3–6] and with the recent treatment of the authors' case by O'Brien [7].

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Authors' Closure

In comparing our results with those of other investigators it should be recalled that we considered a Couette flow over the cavity and that the Reynolds number is based on the velocity of the moving wall rather than the velocity at the free surface.

Thom and Apelt [8][4] consider a Poiseuille flow over the cavity and obtain a dividing streamline that appears to start at the convex corners but dips lower than our dividing streamline. O'Brien [9, 10] has solved the same problem for both Couette and Poiseuille flow. She comes to the conclusion that "Poiseuille flows over cavities uniformly produce lower dividing streamlines than Couette flows for the same geometry."

The discusser correctly observes that in our case there is a large degree of symmetry at $N_{\mathrm{Re}} = 100$ while Burggraf [11] does not find such symmetry in his cavity flow with a scrapping lid at the same N_{Re}. The discrepancy is readily resolved if we recall that Burggraf's Reynolds number is based on the velocity of the upper cavity surface. Since in our problem the velocity of the cavity surface is approximately four percent of the velocity of the channel wall our Reynolds number of 100 corresponds to Burggraf's N_{Re} of approximately 8 for which case his structure would also be symmetric.

The shear and the velocity at the cavity upper ("free") surface is not constant in our problem (which models a two-dimensional static hole). Therefore, no quantitative comparison can be made between the present problem and investigations that consider constant shear [12] and constant velocity [11–14] at the free surface. Qualitatively, we see the similar flow phenomenon in both problems.

O'Brien [9] should be complimented for her thorough and careful investigation of vortices in viscous shear flows over wall cavities. She considers an identical geometry to ours and obtained solutions for $N_{\mathrm{Re}} = 0$. We can compare her results [15] with our calculations at $N_{\mathrm{Re}} = 1$.

1 In both cases the dividing streamline originates at the convex corners and is concave. The lowest height (h/AR) of O'Brien's dividing streamline is lower than ours; see Table 1 of this Closure.

2 While the flow structure is similar in both cases the actual values differ considerably. The values of $\psi_{\max}$ for two geometries are shown in Table 1 of this Closure.

Table 1

	$\psi_{\max}$		h/AR	
AR	O'Brien	Present	O'Brien	Present
1.0	0.0102	0.0121	0.933	0.962
2.0	0.0105	0.0125	0.9665	0.9875

It should be pointed out that for the scrapping lid problem at $N_{\mathrm{Re}} = 0$ our results agree within 4.5 percent with those of O'Brien. The larger discrepancy in the cavity problem is most likely due to computational difficulties associated with the singular nature of the convex corners.

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Buckling of Composite and Homogeneous Isotropic Cylindrical Shells Under Axial and Radial Loading[1]

G. WEMPNER.[2] The authors have managed to obtain relatively simple buckling modes which fulfill eight different boundary conditions by imposing two simultaneous equations involving the circumferential wavelength and the critical load (or axial length). Simultaneous solutions of the polynomial and transcendental equations are obtained numerically by a procedure of trial and correction. A difficult problem has been treated in a thorough manner and useful numerical results have been given. However, several questions arise.

[3] Numbers in brackets designate References at end of Discussion.

[4] Numbers in brackets designate References at end of Closure.

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