

## Improved Linearized Velocity Profiles for Turbulent Free Shear Layers<sup>1</sup>

**R. H. PAGE.**<sup>2</sup> The authors are to be congratulated on their detailed analytical studies of alternate velocity distributions for use with integral conservation equations. Their analytical results must now be compared with experimental data. The determination of whether a Reichardt-type or a Göertler-type velocity profile is the truest representation must be based on the determination of which gives the best agreement with experimental data. Both profiles represent solutions of linearized equations and there is no reason *a priori* to believe that one linearization method is superior to the other.

The authors' statement that smaller values of  $\eta_m$  indicate a better profile may be misleading. Small values of  $\eta_m$  are necessary for the integral procedure but not sufficient for drawing conclusions about exactness or preciseness. For example,  $\eta_m$  for the similar solution of the zero secondary velocity case can be written as

$$\eta_m = \eta_R - \int_{-\infty}^{\eta_R} [\rho/\rho_\infty] \phi^2 d\eta \quad (1)$$

If we desire  $\eta_m$  to be zero (obviously the smallest possible value) this simply means that

$$\int_0^{\eta_R} d\eta = \int_{-\infty}^{\eta_R} [\rho/\rho_\infty] \phi^2 d\eta \quad (2)$$

which can be rewritten as

$$\int_0^{\eta_R} 0.5[1 - F(\eta)] d\eta = \int_{-\infty}^0 0.5[1 + F(\eta)] d\eta \quad (3)$$

where

$$[\rho/\rho_\infty] \phi^2 = 0.5[1 + F(\eta)] \quad (4)$$

Equation (3) will be satisfied when  $F(\eta)$  is represented by any odd function, such as

$$F(\eta) = 1 - e^{-\alpha\eta^2} \quad (5)$$

$$F(\eta) = \sin \beta\eta \quad (6)$$

$$F(\eta) = \operatorname{erf} \eta \quad (7)$$

Equation (7) represents Reichardt's velocity profile for the incompressible case. The authors have used this expression but it

is obvious that equation (7) is only one of a number of possible profiles which satisfy the condition that  $\eta_m$  equals 0. Thus the fact that  $\eta_m = 0$  merely means that  $F(\eta)$  is an odd function and should not, by itself, be considered a test for exactness of the profile. In fact, the Göertler velocity profile,  $\phi = 0.5(1 + \operatorname{erf} \eta)$ , for which  $\eta_m$  is not zero may be a better representation. Only a careful comparison of the theoretical profiles with experimental data can lead to a determination of which is a "better" profile.

### Authors' Closure

The authors appreciate Professor Page's very interesting comments concerning the relative merits of the two types of linearized profiles. We agree with his statement that there is no reason, *a priori* to favor either type of profile. His arguments are concerned largely with the effect of linearization on profile shape. This is a valid inquiry which should be explored further, especially as regards the choice of a basic distribution, i.e., error, Gaussian, or trigonometric functions.

The primary thesis of our presentation was that, inasmuch as the type of linearization appears to have a relatively small effect on profile shape, Fig. 4, the location of the profile in space, as characterized by the integral shift parameter  $\eta_m$ , is of equal importance in determining the suitability of a given type of profile for use in an integral analysis. In this regard the Reichardt-type profiles were shown to be superior to the Oseen distributions. We did not mean to imply that, because of their lower  $\eta_m$  values, the former profiles were "better" in all respects; however, based on the foregoing reasoning, we did conclude that they were "better representations of the flow."

The question to be answered by an individual investigator, when contemplating the alternative profiles, is whether the simplicity afforded by elimination of the shift is offset by the possibility that the inaccuracy due to profile shape *might* be increased slightly. This will, of course, remain a judgment decision until further investigation is complete.

## End Effect Bending Stresses in Cables<sup>1</sup>

**ROBERT PLUNKETT.**<sup>2</sup> The author presents a very neat demonstration of how the solution for the elastica can be modified to take care of the boundary conditions of the title problem for

<sup>1</sup> By J. P. Lamb, T. F. Greenwood, and J. L. Gaddis, published in the December, 1969, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 36, TRANS. ASME, Vol. 91, Series E, pp. 657-663.

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<sup>1</sup> By J. A. DeRuntz, Jr., published in the December, 1969, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 36, TRANS. ASME, Vol. 91, Series E, pp. 750-756.

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