discussion viewed the sheet as consisting of an *isotropic* rigid, plastic matrix that is constrained by inextensible fibers. The latter concept appears more natural to the author because, for most composites, the matrix material is isotropic. On the other hand, the equations expressing the former concept contain an additional constant, which is useful in fitting theory to experimental data.

Optimization of a Viscoelastic Structure: The Seat-Belt Problem[1]

RAYMOND R. McHENRY.[2] This paper presents an interesting analytical exercise. However, one of the major shortcomings of optimization studies of this type for the automobile restraint system problem (e.g., a similar simplified study is presented in reference [1][3]) is a tendency toward oversimplification. In the present case, the effects of the deceleration time history of the vehicle, the jackknifing motion of the occupant, belt angularity (side view) and slack, seat cushion deflection and friction, etc., are neglected.

There are in existence several relatively complete nonlinear mathematical models of the automobile crash victim in a longitudinal collision [2, 3] which could be applied in optimization studies with techniques that operate externally of the mathematical model, on the inputs and outputs (e.g., the method described in reference [4]). However, limitations on practical belt material properties and on achievable vehicle deceleration waveforms make the benefits of such a theoretical optimization rather doubtful.

It should be noted that an "optimum" restraint must be applied to wide ranges of occupant sizes and weights, that the interior dimensions of automobiles include wide variations, and that the different obstacles encountered in automobile collisions produce different deceleration waveforms for the vehicle. Also, the criterion for optimum performance must include a weighted sum of several individual responses (i.e., the peak belt load on the pelvic region is not as important a source of injury as are head and thorax impacts on the vehicle interior).

The general effect that is sought with viscoelastic materials in the present paper is the same as that which can readily be achieved with an inertia reel on a conventional belt.

The "severity index" is calculated for the belt loading, whereas the cited "critical value" of 1000 is related to head impact on the vehicle interior.

The cited 8 mph impact velocity threshold for windshield penetration was presented in the source document as a speed below which the head of an *unrestrained* occupant does not hit the windshield. With a belted occupant, the head impact velocity on the vehicle interior is substantially greater than the pelvic velocity (i.e., jackknifing motion occurs). Therefore, the use of an 8 mph limitation on pelvic velocity, at a specified forward displacement, is difficult to justify.

The numerical examples indicate an occupant mass corresponding to a weight of 180 lb. It is well established [5] that only approximately 60 percent of the mass of an occupant is decelerated by a lap belt.

Finally, the synopsis tends to be confusing because of the fact that constraints (*a*) and (*b*) are not compatible with each other, whereas each is applied in combination with constraint (*c*).

It is unfortunate that the cited difficulties, related to oversimplification of the particular application selected by the authors, detract from the excellent quality of both the analytical study and the technical presentation.

References

1 Kaufman, H., and Larson, D. B., "Calculation of Deceleration Waveforms Using Optimal Control Theory," *Proceedings of the First International Conference on Vehicle Mechanics*, Wayne State University, July 16–18, 1968.

2 McHenry, R. R., and Naab, K. N., "Computer Simulation of the Crash Victim—A Validation Study," Tenth Stapp Car Crash Conference, November 8–9, 1966.

3 Chaffin, D. B., "A Computerized Biomechanical Model—Development and Use in Studying Gross Body Motions," Technical Conference on Biomechanics, University of Michigan, June 12–13, 1969.

4 Fogerty, L. E., and Howe, R. M., "Trajectory Optimization by a Direct Descent Process," NASA CR-1070, June 1968.

5 Ryan, J. J., "Reduction in Crash Forces," *Proceedings of the Fifth Stapp Conference*, University of Minnesota, 1962.

L. W. Morland.[4] This paper makes a preliminary investigation of viscoelastic creep (relaxation) as a feature of seat-belt design. The model adopted is a point mass attached to the midpoint of a tension supporting belt fixed at both ends, and assumes small displacement and infinitesimal strain in the belt during the motion caused by a suddenly imparted velocity (change) to the mass. Two element (Maxwell and Voigt) models are considered to describe the viscoelastic properties, and quasi-static motion is assumed; for the given geometry wave travel times over the belt length are a factor 10^{-2} of the motion time. The initial velocity is maximized subject to restrictions on the final displacement and velocity, and maximum force exerted on the mass during the motion. While the displacements considered are not compatible with the linearity assumption in the geometry and the strain, so that the solution detail is not useful, the qualitative effect of viscoelasticity in comparison with purely elastic results indicates the value of further investigation. More realistic viscoelastic behavior and description of the body motion should be feasible, perhaps incorporating a double-belt arrangement for which displacements of the "main body" can be kept small.

Authors' Closure

The authors are grateful to Professor Morland and to Mr. McHenry for their comments on our paper. We recognize the complexity of the actual problem of design of automobile passenger restraint systems, but the paper was definitely not intended to be a study of such a system, as indicated by our choice of words: ". . . .highly idealized problem. . . . ." Rather, it is to show that viscoelastic properties of the belt material can be considered as additional parameters in the optimization, and that significant improvement in belt performance can be obtained by such consideration. Other optimization studies that we have seen have not included viscoelastic material parameters in the analysis. Since this is an initial study, a much-simplified geometry and material description was chosen to make the central point, and data, for the numerical examples in Table 1, then had to be chosen to be representative of the numbers that we found in the references rather than to be precise. For example, a point mass, which is the model chosen in the paper, cannot represent simultaneously the head and the pelvis, and the effect of jackknifing motion cannot properly be taken into account in the numerical example.

With regard to the choice of the 8 mph impact velocity threshold for windshield penetration, on which McHenry commented, we have the following response. It follows from the cited reference [6] that, for sled velocities below 8 mph, the head of an

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[1] By W. Nachbar and J. B. Schipmölder, published in the September, 1969, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 36, TRANS. ASME, Vol. 91, Series E, pp. 565–572.

[2] Head, Engineering Mechanics Section, Transportation Research Department, Cornell Aeronautical Laboratory, Inc., Buffalo, N. Y.

[3] Numbers in brackets designate References at end of Discussion.

[4] Affiliation and location unknown.

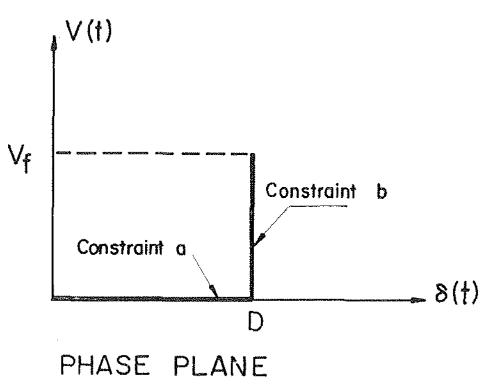

Fig. 1

unrestrained passenger does not reach the windshield. This is an estimate of the maximum allowable V_0 (= sled velocity) if our requirement were that an unrestrained passenger is not to reach the windshield. But that is not our requirement in the paper. The problem posed was to maximize V_0 without restriction on V_0 itself, and the requirement which we did use was that if the passenger hits the windshield, this should occur at a head-to-glass impact speed (HGIS) which is smaller than or equal to a given critical speed which we called V_f. From the captions in the photographs on p. 162 of the cited reference [6], it follows that 8 mph is not only a safe sled speed but is also a safe HGIS. For HGIS less than 8 mph, the head does not penetrate the windshield. This conclusion appears to us not to depend on whether or not the passenger is restrained. Hence our choice of V_f = 8 mph in the examples. These examples were included only to suggest that representative numerical cases would give reasonable results. The main results of the paper are in nondimensional form and are not affected by the particular data in Table 1.

Moreover, the paper does point out (see Fig. 4) that belt performance close to the optimum in this simplified problem is obtained over a wide range of conditions and material properties chosen around the optimizing values. If this relative insensitivity continues to hold for more realistic studies, then this would tend to remove the criticism of the "optimum" restraint that McHenry gives in the third paragraph of his comments.

The conventional inertia reel locks at accelerations higher than a critical value; the performance of such a device would be generally different from viscoelastic effects. An inertia reel which would be represented schematically as a spring and dashpot arrangement could simulate the elementary viscoelastic models used in this paper, but, it would appear, at the cost of greater mechanical complexity when more restraint than just a lap belt is considered. We cannot imagine that more realistic representations of viscoelastic behavior, e.g., a general relaxation modulus for a linear viscoelastic material, could be achieved with an inertia reel. The general relaxation modulus offers promise of providing the totally different behavior at different strain rates that we have proposed to be desirable for the passenger restraint system.

Constraints (a) and (b) of the abstract are abbreviated descriptions of constraints (1a–c) and (1e–g) of the paper. They are consistent and merely state conditions along different parts of the same boundary in the phase plane. In Fig. 1 of this Closure, the terminal point at $t = t_f$ of the solution must lie along the heavy lines shown in the plane of $V(t)$ versus $\delta(t)$. The horizontal line is constraint (a); the vertical line is constraint (b).

The comments just given are not offered in any way to disagree with the main thrust of the discussions of Morland and McHenry. Indeed, these discussions offer positive direction for future study in this area.

Elastic-Plastic Deformation at Finite Strains[1]

S. NEMAT-NASSER.[2] Professor Lee, in his usual manner, has presented in his paper[1] a very interesting and sound theory of elastic-plastic deformation at finite strains. In this theory, large hydrostatic pressures, such as can occur in explosive forming, are envisaged, resulting in finite elastic strains which are predominantly dilatational. Such essentially dilatational elastic strains lead, of course, to finite changes in volume (or mass-density) that must be taken into account in the theory. There is evidence that large hydrostatic pressure does affect the yield condition and the hardening phenomenon, as is shown by Bridgeman.[3] This may be essentially because hydrostatic pressure induces an increase in the dislocation density and, as is pointed out by Professor Lee, "the production of a prescribed dislocation density associated with a prescribed state of hardening is likely to require more plastic work under high pressure rather than less . . . ." It is also likely that hydrostatic pressure hinders the motion of dislocations, requiring still more plastic work. And, of course, under a very high pressure much more plastic work is required, resulting in a hardening phenomenon that is dependent on the hydrostatic pressure.

The theory developed in the author's paper, therefore, becomes more relevant if it also accounts for the effect of hydrostatic pressure, since it envisages pressures of such magnitude that can induce finite elastic dilatation in metals. Moreover, the inclusion of such an effect would eliminate certain contradictions which are otherwise involved. For example, in the author's paper it is assumed that the yield condition for finite elastic-plastic strains has the form

$$f\left(\frac{\rho_0}{\rho}\,\mathsf{T}\right) = c \qquad (1)$$

rather than the natural form

$$f(\mathsf{T}) = c \qquad (2)$$

where T is Cauchy's (or the true) stress tensor, and ρ_0 and ρ are the initial and the current mass-densities, respectively. Professor Lee, because of his initial postulate that plastic flow is unaffected by hydrostatic pressures, is compelled to employ (1), rather than (2), so as to possibly remove the contradiction of less work under hydrostatic pressure, see equation (27) in the author's paper.

Of course assumption (1), which is used in the author's paper, contradicts the postulate that plastic deformation is not influenced by hydrostatic pressure unless the material is incompressible, in which case no *dilatation* can take place.

To cope with the above-stated difficulties, I suggest for the assumed isotropic case, the following modified yield condition, which includes (1) as a special case, and which accounts quite naturally for the effect of the hydrostatic pressure

$$f(J_2', J_3') = c(\varphi, \theta, tr\,\mathsf{T}), \qquad (3)$$

where J_2' and J_3' are the second and third invariants of the Cauchy stress deviator, $\mathsf{T}' = \mathsf{T} - \frac{1}{3}(tr\,\mathsf{T})\,\mathsf{I}$, I being the identity

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[1] By E. H. Lee, published in the March, 1969, issue of the JOURNAL OF APPLIED MECHANICS, Vol. 36, TRANS. ASME, Vol. 91, Series E, pp. 1–6.

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[3] Bridgman, P. W., *Studies in Large Plastic Flow and Fracture*, Harvard University Press, Cambridge, Mass., 1964, Chapter 2.