this becomes

$$\mu_n' = x_o^n \int_0^\infty \eta^{n/m} \exp(-\eta) d\eta \ldots\ldots\ldots\ldots [4]$$

This integral can be expressed in terms of the Gamma function

$$\mu_n' = x_o^n \, \Gamma(1 + n/m) \ldots\ldots\ldots\ldots [5]$$

The second and third moments about the mean are

$$\mu_2 = x_o^2[\Gamma(1+2/m) - \Gamma^2(1+1/m)]$$

and $\mu_3 = x_o^3[\Gamma(1+3/m) - 3\Gamma(1+2/m)\Gamma(1+1/m)$
$$+ 2\Gamma^3(1+1/m)] \ldots\ldots\ldots [6]$$

From these a measure of the skewness can be obtained

$$\alpha_3 \equiv \mu_3/\mu_2^{3/2} \ldots\ldots\ldots\ldots [7]$$

Since α_3 is a function of m only, the value of m can be chosen so that the values of α_3 for the theoretical distribution and the experimental data coincide. Then since the second moment about the mean, that is, the square of the standard deviation, of the experimental data is known, the relation

$$\mu_1/x_o^2 = \Gamma(1 + 2/m) - \Gamma^2(1 + 1/m) \ldots\ldots\ldots [8]$$

can be solved for x_o. Finally, the relation

$$\mu_1'/x_o = (\bar{x} - x_u)/x_o = \Gamma(1 + 1/m) \ldots\ldots\ldots [9]$$

can be solved for x_u, since the mean of the experimental data is known. Plots of the quantities α_3, μ_2/x_o^2, and μ_1'/x_o are given in Figs. 1 and 2 of this discussion.

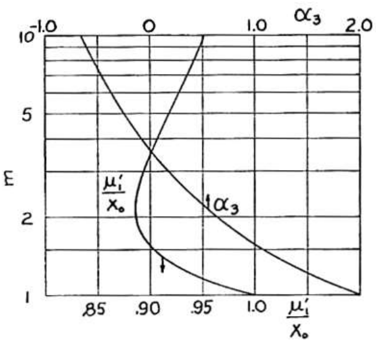

Fig. 1

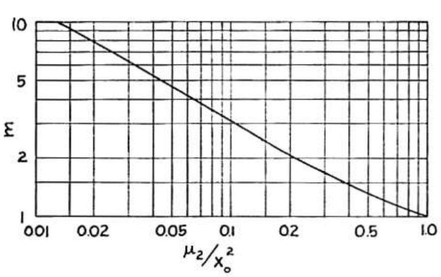

Fig. 2

The writer would like to ask what procedure, preferably systematic, should be followed in the case of a "complex" distribution. An extension of the foregoing procedure looks impractical, and yet the writer would like to try applying the distribution in other cases. For example, it would be interesting to see whether the other data on the ST-37 steel reported by Müller-Stock would result in the same division of the population as found from Figs. 6 and 7 of the paper.

Author's Closure

The author appreciates the comments made by the discussers. The proposal of Professor Tsu to take any three sets of the values P and x is quite correct but does not use the data efficiently. This method may be improved by taking the set from a smoothed curve. Up to the past year the author's usual method has been to plot the data as shown in the paper and to choose the value x_u to give the best straight line. In this way it is easy to decide if the distribution is simple or complex, but the procedure is not entirely free of subjectiveness.

About a year ago the author decided that it would be better to start by standardizing the variable x, i.e., by putting $z = (x - \bar{x})/\sigma$, where $\bar{x}$ is the mean and σ the standard deviation and eliminating two of the parameters, for instance, x_u and x_0. The distribution function then takes the form

$$P = 1 - \exp\{-[z\sqrt{\pi(2\alpha) - \pi^2(\alpha)} + \pi(\alpha)]^{1/\alpha}\}$$

where $\alpha = 1/\text{m}$.

A curve paper for different values of α, also including the standardized Gaussian distribution, may be prepared. By plotting the points (P, z) on this paper, it is easy to decide whether the distribution is simple or complex and to estimate, with a good approximation, the value of α.

As to the third question, the parentheses are an awkward misprint. The values for $\log(x - x_u)$ in Fig. 2 do not correspond to the given value $x_u = 1.5 \times 20\mu$ but to $x_u = 1 \times 20\ \mu$. It should be mentioned that the x-values are mid-point values and should correctly have been increased by $^1/_2$. Thus the value $x_u = 30\ \mu$ is the correct one.

The introduction of a maximum value x_m proposed by Mr. Mugele is a valuable extension of the function. It was not found necessary to introduce this new parameter in the field of strength of materials, probably because the theoretical strength may be perhaps a hundred times higher than the technical strength. But in other fields conditions may be quite different.

The method proposed by Professor McClintock to use the first three moments is quite good if the distribution is simple and the population not too small. The author has been aware of this possibility of computing the parameters and has mentioned it (with some different notation for the gamma function) in an earlier paper.[8] Actually, however, he has never applied this method, but admits that it may sometimes have its advantages.

As to the question of a systematic procedure when the distribution is complex, the author is sorry to admit that so far he has found no better method than to cut and try. This is, of course, not very satisfactory, but a simple electronic computing machine, recently completed, facilitates the otherwise tedious computations.

The Theory of Plasticity Applied to a Problem of Machining[1]

B. T. Chao[2] and K. J. Trigger.[3] In applying the new method of analyzing stress and strain distributions during chip formation, the authors assume that in commercial high-speed machining operations the metal behaves like an ideally plastic

[8] "The Phenomenon of Rupture in Solids," by Waloddi Weibull, IVA Handling, No. 153, p. 23.

[1] By E. H. Lee and B. W. Shaffer, published in the December, 1951, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 73, pp. 405–413.

[2] Assistant Professor of Mechanical Engineering, University of Illinois, Urbana, Ill.

[3] Professor of Mechanical Engineering, University of Illinois. Mem. ASME.

material that does not work-harden. Experience obtained by the writers in the actual cutting of three kinds of steel and one type of electrolytic copper fully justifies such an assumption. In fact, this constitutes one of the major conclusions reached by one of the writers 4 years ago after conducting research in metal cutting in the Machine Tool Laboratory of the University of Manchester.[4] A ductile material undergoing high-speed chip formation acts as if no work-hardening occurred during the time that deformation takes place along the shear surface as indicated by line AC in Fig. 1 of the paper. However, this does not mean that the deformed metal (chip) is not strain-hardened. It was first shown by E. G. Herbert that the hardness of the chip may be several times that of the undeformed chip material. That such a paradox can be explained by the dislocation theory for the plastic deformation of crystalline solids has been given in a recent paper.[5]

It should be pointed out that the yield stress k referred to by the authors is the dynamic yield stress. It differs appreciably in magnitude from either Tresca's yield limit or von Mises' quadratic limit. Both these quantities refer to quasi-static loading conditions. For structural steels, it has been found that the dynamic yield stress is in the neighborhood of the static ultimate tensile strength.

The authors indicate that the influence of the compressive stress on the dynamic yield stress is much less than that assumed by Merchant. Recently this has been brought out in the cutting investigation of spheroidized SAE 52100 steel using a wide range of speeds, feeds, and three different tool-rake angles. No correlation has been found to exist between the so-called machining constant and the "assumed" slope of yield stress-compressive stress curve. However, Merchant's plasticity equation does agree with experimental data to a good degree of approximation

$$2\,(\text{shear angle}) + \left(\begin{matrix}\text{friction angle at}\\ \text{tool chip interface}\end{matrix}\right) - \text{tool rake} = \text{const}$$

The authors' speculation of the existence of a tiny built-up edge which arises naturally as a consequence of the mathematical analysis may serve as an explanation for the foregoing controversy and should be explored fully by future experimentation.

Except for the small percentage which is retained as latent heat in the deformed chip, nearly all of the energy expended in metal-machining operations transforms into sensible heat, resulting in a large temperature rise. The effect of such temperature rise on the mechanism of chip formation has been reported recently.[5,6] In distinction to what the authors have stated, the dynamic yield stress is practically unaffected, if shearing takes place in a narrow zone, since the heating effect does not actually occur until after the shear energy has been imparted to cause plastic deformation. On the other hand, such heating due to deformation and that produced due to tool-chip rubbing do have an appreciable effect on the mechanism of chip formation through their influence on interface friction.

With a slip-line field configuration as shown in Fig. 6 of the paper, the deformation process is greatly complicated. Shearing takes place over a fan-shaped region AFC. Both temperature and strain-hardening may exhibit their influence, depending not only on the magnitude of temperature and strain involved but also upon the effective time of heating. It is difficult to analyze this aspect of the built-up nose solution without further experimentation.

Quantitative verification of the authors' new theory needs an accurate determination of the tool-chip friction angle λ_t which is not the same as that determined under the condition when shear occurs over a narrow band. It cannot readily be calculated from tool-force dynamometer data and chip thickness measurements. Examination of the nature of tool-chip contact under a magnification of $\times 30$ to $\times 50$ after the tool has been in use for a short time reveals the existence of two distinct regions of contact on the active tool face. This may serve as a clue for the experimental determination of the angle θ by actually measuring the length EC in Fig. 6. Once θ is known, together with the tool-force and chip-thickness data, the combined Mohr circle diagram, Fig. 8 of the paper, can be drawn without difficulty.

It is hoped that the authors will continue their work in the future.

R. S. Hahn.[7] This paper presents an interesting analysis of the metal-cutting process and seems to be the first to give an analysis involving the built-up edge. Previously it has been considered that the type 2 chip was formed without a built-up edge. The writer, in tests where the chip appeared to be of type 2, has found a very small built-up edge in all cases—indeed a few thousandth of an inch in size. (Chips that are produced by an abrupt ending caused by the tool entering an interruption often carry at their end a very small built-up edge which can be seen under the microscope.) Such chips, even though their back side is very smooth and highly burnished, have been formed with a built-up edge. The region BC in Fig. 6 of the paper serves to burnish the chip so highly that all roughness caused by the flow along FC is eradicated, and generally it has been believed that such chips were produced without a built-up edge. Consequently, it appears that there is considerable truth in the authors' conclusion that a built-up edge always exists.

The analysis presented considers flow to take place in the v-direction only, Fig. 6. That this is not strictly true is evidenced by the curvature of the chip. The authors state that the curva-

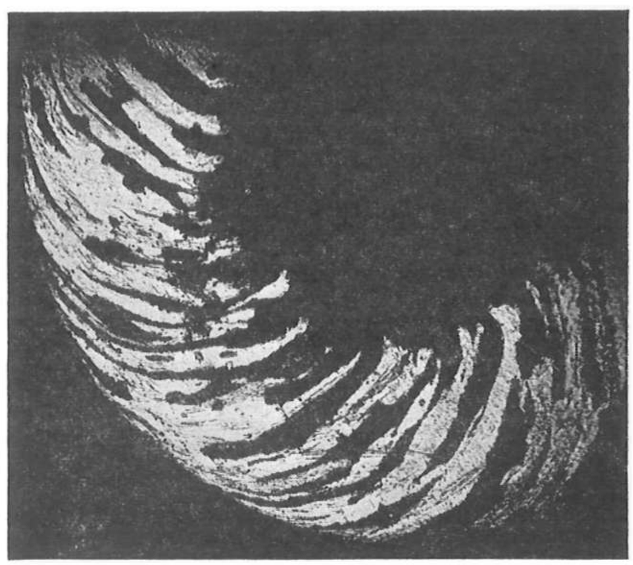

Fig. 1 Photomicrograph of Chip Showing Curved Elongated Grains Found in Chips of Small Radius; $\times 200$

[4] "On Elements in Metal Machining-Dynamic Factors in Orthogonal Cutting, by B. T. Chao, 1947, PhD Dissertation, Library of the College of Technology, University of Manchester.

[5] "Thermophysical Aspects of Metal Cutting," by B. T. Chao, K. J. Trigger, and L. B. Zylstra, presented at the Annual Meeting, Atlantic City, Nov. 25–30, 1951, of The American Society of Mechanical Engineers.

[6] "Cutting Temperatures and Metal-Cutting Phenomena," by B. T. Chao and K. J. Trigger, Trans. ASME, vol. 73, 1951, pp. 777–793.

[7] Research Engineer, The Heald Machine Company, Worcester, Mass. Mem. ASME.

ture of the chip is due to residual stress and thermal strain conditions. The fact that curved chips may be heated to incandescence without changing curvature tends to disprove the foregoing. Also, they will be found to retain their curvature to the last as they are dissolved in acid. It appears from experiments that the chip curvature is a result of flow along the u-slip lines as well as the v-slip lines. In certain cases the chip velocity of the metal adjacent the tool face has been observed to be as much as 30 per cent higher than the velocity of the metal forming the outside face of the chip. We may consider the grains in the uncut metal to be essentially equiaxed so that after deformation they become elongated showing the directions of principal strains. It will be evident from Fig. 1 of this discussion that a nonhomogeneous state of strain exists and that the authors' analysis will not apply accurately in such cases.

M. Eugene Merchant.[8] This paper marks another important step forward in the development of the science of metal cutting. However, while the present-day plasticity theory used by the authors is sound and well established, it appears that the practical assumptions made in applying it to the metal-cutting problem are open to some question. The result is that while the qualitative findings of the resulting limiting chip-stress solution agree well with observed facts and in some cases offer a better explanation of these facts than had previously existed, nevertheless the quantitative predictions of that solution do not agree as well with experimental data as do the predictions of the more approximate minimum-energy solution.

The first case which the authors consider is that of machining without a built-up edge. In applying the theory to this case they assume that the yield stress is a constant, yet it is known that this stress is affected by a number of factors. Among the more important of these is that of normal stress (hydrostatic pressure), the effect of which has been demonstrated by Bridgman[9] and Rotner[10] (the writer is indebted to Prof. M. C. Shaw of M.I.T. for his kindness in bringing the latter reference to his attention). Measurements made in our own laboratory by Kemeny[11] and by Krabacher and Whisler[12] on a Bridgman-type apparatus confirm this same effect. Data obtained in these investigations, as presented by Ernst[13] are shown in Fig. 2 of this discussion. Introduction of this effect of normal stress on yield stress into the minimum-energy solution has a profound effect upon that solution, making it a much better approximation to reality. Therefore is it not likely that the introduction of this same effect into the authors' limiting chip-stress solution would also have a profound effect on the quantitative results?

The authors consider next the case of machining with a built-up edge. The fact that the authors' limiting chip-stress solution predicts the presence of a built-up edge marks a real step forward in the mechanics of cutting. Little progress has been made in developing the mechanics of this case previously. However, the quantitative predictions of the theory as to the size of the built-up edge, the values of friction at which it comes into existence, and so on, are questionable because of the assumptions made. Primarily, yield stress again is assumed to be a constant and independent of the several factors which are known to influence it. This assumption alone could have a profound effect on the quantitative results of the theory as already indicated. Then, too, the authors assume that the bottom of the built-up edge is in frictional contact with the work surface, whereas photomicrographs of chip formation have shown that the built-up edge actually is continuous with the work material at this point. Continuous plastic flow takes place in this region in the same manner as it does on the shear plane and strain-hardening also occurs. As a result the bottom portion of the built-up edge gradually builds downward, increasing in size, until it becomes so large as to be unstable and is then carried away with the machined surface, after which the cycle begins again. (It is this process of the building up and sloughing off of fragments of built-up edge on the machined surface which accounts for the roughness of such surfaces when produced under condition where a built-up edge exists.)

In the composite solution, resulting from the consideration of the two cases already discussed, the authors have presented a comparison of the quantitative predictions of the limiting chip-stress solution with experiment in Fig. 12 of the paper. They indicate that the reason the experimental data extend beyond the theoretical boundary determined by $\mu_t = 1$ in the figure is that elastic deformation modifies the solution at the tool-chip interface. If this is so, then it would appear that the data should show at least some downward trend toward point F in Fig. 12 beyond the boundary, in keeping with the downward trend of the boundary. However, no such downward trend is observed in these data nor even in other data obtained at higher values of λ-γ than those shown experimentally in Fig. 12 of the paper. On the other hand, the minimum-energy solution, though more approximate from a theoretical point of view, actually gives a better approximation to the experimental results than that shown for the limiting chip-stress solution in Fig. 12. The degree of approximation given by the minimum energy solution is illustrated in Fig. 3 of this discussion. (The symbols used are those originally proposed by the writer,[14] but the ordinate and abscissa are the same as those of the authors' Fig. 12.)

It can be seen that the degree of approximation of the experimental data to the line labeled $2\phi + \tau - \alpha = 77$ deg predicted by the minimum-energy solution, is rather better than the approximation predicted by the limiting chip-stress solution in Fig. 12. However, the authors object to this agreement since it involves the conclusion that the proportionality constant relating yield stress to normal stress must be of the order of 0.23 (cot 77 deg), since according to the minimum-energy solution the sum $2\phi + \tau - \alpha$ should equal approximately the complement of the slope angle of the yield stress versus normal stress line in a plot such as that shown in Fig. 2 of this discussion. Nevertheless, Bridgman,[9] Rotner,[10] Kemeny,[11] and Krabacher and Whisler[12] all obtain values of this order for the proportionality constant for a variety of metals. For example, a comparison between the values of $2\phi + \tau - \alpha$, obtained from metal-cutting tests, and values of the complement of the slope angle of the yield-stress plot, obtained from torsion-compression tests made in this laboratory on four different steels, is given in Table 1. It can be seen that the degree of correlation between the data obtained from the two quite different types of tests is relatively good. It would appear therefore that for the values of strain common in metal cutting the influence of hydrostatic pressure on yield stress is of a large enough magnitude to be significant in the theory of machining.

Since the authors' limiting chip-stress solution based on plasticity theory actually results in a poorer approximation to experiment than does the simple and approximate minimum-energy

[8] Assistant Director of Research, The Cincinnati Milling Machine Company, Cincinnati, Ohio. Mem. ASME.

[9] See Authors' Bibliography, reference (19).

[10] "Change of Mechanical Properties of Metals Under Hydrostatic Pressure," by S. I. Rotner, *Journal of Technical Physics*, USSR, vol. 19, 3rd edition, March, 1949, p. 408.

[11] "Torsion and Compression Testing," by J. Kemeny, University of Cincinnati thesis, 1947.

[12] "Torsion and Compression Testing" by E. J. Krabacher and K. W. Whisler, University of Cincinnati thesis, 1949.

[13] "Fundamental Aspects of Metal Cutting and Cutting Fluid Action," by Hans Ernst, Annals of the New York Academy of Sciences, series II, vol. 53, 1951, pp. 805–823.

[14] See Authors' Bibliography, reference (18).

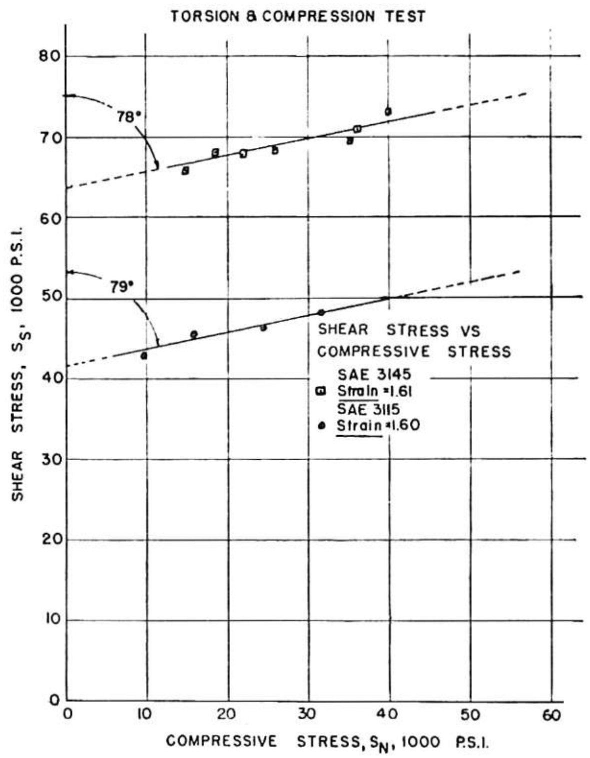

Fig. 2

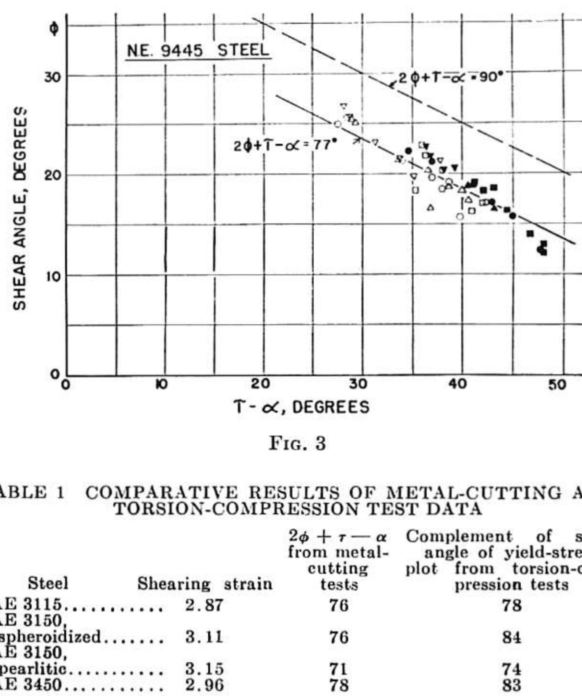

Fig. 3

TABLE 1 COMPARATIVE RESULTS OF METAL-CUTTING AND TORSION-COMPRESSION TEST DATA

Steel	Shearing strain	$2\phi + \tau - \alpha$ from metal-cutting tests	Complement of slope angle of yield-stress plot from torsion-compression tests
SAE 3115...........	2.87	76	78
SAE 3150, spheroidized.......	3.11	76	84
SAE 3150, pearlitic...........	3.15	71	74
SAE 3450...........	2.96	78	83

solution, even though on a theoretical basis it should be expected to be more exact, it would seem that the next desirable step for the authors to take would be the introduction of improved assumptions into their solution to determine the extent to which these help to bring it into line with experimental findings. Furthermore, it would appear that the authors would benefit by carrying out a few critical experiments to test certain points in their theory as a guide in choosing the assumptions to be used.

It is the writer's feeling that if these things are done a useful and practical theory can be developed from this approach which will mark a major advance in the science of machining.

E. K. Henriksen.[15] Metal-cutting operations have been investigated more or less continuously since the 1870's and, therefore, it is not surprising that it is hard to find basically new thoughts in the constant flow of contemporary publications on this subject. The present, paper, however, is a very pleasant exception to this rule. The authors have approached the problem by original methods, and they have laid a new cornerstone to the foundation of a great, yet unfinished building.

However, many engineers and other workers in the metal-cutting field will fail to visualize the value and importance of this paper, simply because they do not feel themselves prepared to follow the mathematical methods used. For this reason the writer would like to present a simplified development, leading up to the principal equation.

Referring to Fig. 4 of this discussion, AC is assumed to be the

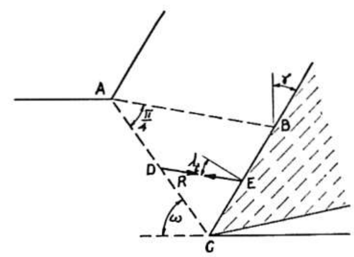

Fig. 4

plane where the shearing deformation occurs, and it follows that it is the plane of maximum shearing stress. B is the upper limit for contact between chip and tool and, therefore, the upper limit for transmission of force between the two bodies. It follows then:

There is no load on AB.
There is no stress on AB.
Specifically there is no shearing stress on AB.
Hence AB is a plane of principal stress.

From the general rule that the maximum shear occurs under 45 deg with the planes of principal stress, it follows that

$$\sphericalangle BAC = 45 \text{ deg}$$

With the shearing stress k and the normal stress p, it can be shown elementarily by means of a triangular free-body diagram $p = k$.

Now the shear stress is contant, equal to k, over AC; it is reasonable to assume that also the normal stress is constant, and equal to p. But then the resultant R will go through the middle of AC, under an angle of 45 deg and therefore, be parallel to AB and its point of intersection E with BC will be the middle of BC. Equilibrium of the body ABC requires that the resultant of the forces on BC will fall in the same line, that is, pass through E, but as E is the middle of BC it is reasonable (so far, at least, see later) to assume that the load on BC is contant.

Now

$$\sphericalangle DEC = \frac{\pi}{2} - \lambda_t$$

where λ_t is the friction angle between chip and tool, hence (from the triangles ABC and DEC)

$$\frac{\pi}{2} + \gamma - \omega + \frac{\pi}{2} - \lambda_t + \frac{\pi}{4} = \pi$$

[15] Professor in Charge. Head of Department of Materials Processing, Cornell University, Ithaca, N. Y. Mem. ASME.

and

$$\omega = \frac{\pi}{4} - \lambda_t + \gamma$$

which is the basic Equation [2] of the paper. The rest is elementary.

As to the subject itself, the authors have based their analysis upon the assumption of no load beyond point B, that is, ignoring elastic stresses in the chip. For analysis of this character such an assumption is natural and justified, the consequences of this assumption being that AB is a straight line, and constant pressure over BC.

The writer would like to show the effect of omitting this limiting assumption.

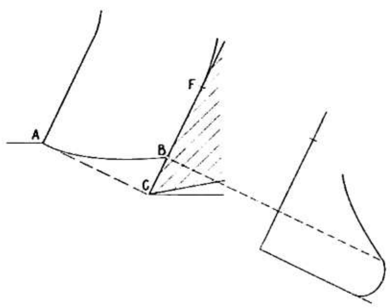

FIG. 5

As shown before,[16] there will be a region FB, Fig. 5 of this discussion, with elastic stresses in the chip and there will be a sharp rise in pressure from F to B. The plastic deformation begins at B and beyond B the rate of increase in pressure will be less, the pressure will reach a maximum and will then decrease somewhat toward C.

The boundary AB between the elastic and the plastic region in the chip itself no longer will be a straight line, but will be curved with the concave side in the direction of the flow of the chip.

AUTHORS' CLOSURE

The authors first wish to express their thanks for the interest shown by the discussers of this paper.

They appreciate the comments of Professors Chao and Trigger confirming the use of an ideally plastic stress-strain relation, and emphasizing the smaller influence of hydrostatic pressure than had been assumed in earlier work. The discussion of the influence of temperature distribution will be important in the future development of the theory; in the analysis under discussion such influences were merely averaged and results compared which would not be expected to be sensitive to such averaging.

We appreciate the comments of Dr. Hahn concerning the experimental observation of a built-up edge as predicted in the paper. We agree that a more detailed study of chip strains including bending effects is called for. In this connection B. W. Shaffer[17] has developed a theory of the turning process with relative rotation between the tool and the work, in contrast to the planing-type theory under discussion. In this solution varying shear through the chip thickness occurs.

Dr. Merchant's discussion is concerned mainly with the influence of hydrostatic pressure on the yield stress in shear. In contrast to the references cited by Dr. Merchant, the discussion below indicates that the consensus of opinion on this controversial issue appears to be against an appreciable hydrostatic-pressure effect.

The study of plasticity of metals has led to the general conclusion that while fracture conditions are highly influenced by hydrostatic pressure, this does not affect plastic-flow conditions appreciably. This is emphasised by the general acceptance of the Mises or Tresca yield conditions, both for initial yield, and as a basis for a generalized stress-strain relation over a range of strain. Both these limits are independent of the average hydrostatic pressure, depending only on the stress deviator. The insensitiveness of the flow stress in tension to hydrostatic pressure up to large strains has been checked directly by Bridgeman.[18] This is in agreement with the concept of plastic potential (see Hill[19]) which has recently received both experimental[20] and theoretical[21] justification. Thus we must answer the question: In metal forming theory why is it generally accepted that hydrostatic pressure affects fracture but not plastic flow, and yet in machining theory an appreciable influence on plastic flow has been commonly accepted? A recent discussion of this question in connection with a paper by Chao and Bisacre[22] shows that many workers in the field of metal-cutting theory do not accept an appreciable influence of hydrostatic pressure. Discussers J. M. Lickley, M. C. Shaw, and E. H. Lee independently emphasize this point. Shaw states: "Calculations based upon atomic structure and inter-ionic forces revealed that normal stresses of about the magnitude that were found in metal cutting should have no influence upon flow stress. That the yield stress for ductile metals was found to be the same in tension and compression was further evidence that normal stress did not influence the ductile properties of metals. Schmidt and Boas had demonstrated that normal stress did not affect the flow stress of ductile single crystals." In the closure Chao and Bisacre agreed with Shaw's comments and mentioned an independent check of Bridgeman's compression-torsion-type test with α brass[23] which gave a zero hydrostatic-pressure effect. In view of these comments, what then is the significance of the papers quoted by Dr. Merchant? In the report,[24] of which the paper under discussion is a shortened version, the shortcomings of the notched torsion-compression tests were discussed. While this test serves admirably to demonstrate the influence of hydrostatic pressure to inhibit fracture, the complex stress and strain distribution makes interpretation in terms of flow stress extremely difficult. This test does not produce shear under hydrostatic pressure, but appreciable longitudinal and lateral strain also which may have an appreciable influence on the results. In machining, such strains are inhibited by the constraint of elastic material, indicating superposed hydrostatic pressure in contrast to the axial compression occurring in the test. It seems therefore that Bridgeman's results for tension quoted above should take

[16] "Stress Distribution in a Continuous Chip—A Solution of the Paradox of Chip Curl," by E. K. Henriksen, Trans. ASME, vol. 73, 1951, pp. 461–466.

[17] "Analysis of Chip Formation in the Turning Operation," by B. W. Shaffer. Brown University Technical Report A11-56, 1951.

[18] "Studies in Large Plastic Flow and Fracture," by P. W. Bridgeman, McGraw-Hill Co., Inc., 1952, p. 64 ff.

[19] "A Variational Principle of Maximum Plastic Work in Classical Plasticity," by R. Hill, *Quarterly Journal of Mechanics and Applied Mathematics*, 1. 1. March, 1948, p. 21.

[20] "A Connexion Between the Criterion of Yield and the Strain Ratio Relationship in Plastic Solids," by G. I. Taylor, Proceedings of the Royal Society, series A, vol. 191, 1947 p. 441.

[21] "Some Implications of Work-Hardening and Ideal Plasticity," by D. C. Drucker, *Quarterly of Applied Mathematics*, 7, 9, 1950, p. 411.

[22] "The Effect of Feed and Speed on the Mechanics of Metal Cutting," by B. T. Chao and G. H. Bisacre. Proceedings of The Institute of Mechanical Engineers, 65(W.E.P. No. 63), 1951.

[23] Report MF/MS 37 to the Mechanical Engineering Research Board of the Department of Scientific and Industrial Research. G. H. Bisacre, 1950.

[24] "The Theory of Plasticity Applied to a Problem in Machining," by E. H. Lee and B. W. Shaffer. Brown University Technical Report A11-43, 1949.

precedence over his results in this test. Because of the language difficulty we were unable to examine Rotner's work in detail in the short time available.

Thus it seems that the agreement of experimental results with Merchant's theory including a pressure effect calls for explanation on other grounds. This point was made by J. M. Lickley in the discussion mentioned previously. The limiting chip stress solutions given in the paper under discussion offer such an explanation.

The discussion of built-up edge development given by Dr. Merchant corresponds to a large intermittent nose usually classed as a type-3 chip. Perhaps the small built-up nose discussed in the paper behaves differently according to its connection with the base material. However, it would be of interest to study the influence of material shear under the nose base on the present theory. We do not appreciate the need for a tendency toward the point F in Fig. 12 of the paper, as suggested by Dr. Merchant. It would seem that elastic effects might very well merely involve smooth extension of the curves μ_n = constant beyond the boundary $\mu_t = 1$.

We agree that the simple derivation given by Professor Henriksen may have greater appeal to some engineers. In the case of the built-up nose solution it seems that it may not be possible to avoid the mathematical considerations in this way. We agree with the importance of considering the bending effect in a more detailed analysis.

The Calculated Performance of Dynamically Loaded Sleeve Bearings—III[1]

G. B. Du Bois.[2] We have some experimental data which is related to this paper, as it tends to support the short-bearing approximate method of solution for the steady-load case. The theoretical part of our project by Professor Ocvirk is listed in the author's Bibliography as reference (12).

Prof. F. W. Ocvirk and the writer are associated with a research project at Cornell University being sponsored and financed by the National Advisory Committee for Aeronautics. This research project was begun in 1948, and at the present time is engaged in an experimental investigation of eccentricity ratio, friction, and oil flow to obtain actual data on this subject for the steady-load case. The short-bearing approximation theory for the steady-load case was extended recently by Professor Ocvirk and is being used as a convenient vehicle for plotting the experimental results. Both Professor Ocvirk and the writer would like to offer some brief comments.

We are using a $1^3/_8$-in. shaft, at speeds up to 6000 rpm, loads on projected area up to 900 psi, with SAE 10 oil. The eccentricity of the shaft in the bearing is being measured by a system of levers and dial gages, and we also are obtaining friction and oil-flow data in both directions of rotation. At the present time the data cover length-diameter ratios of $^1/_4$, $^1/_2$, and 1.

Using the short-bearing approximate method, theoretical lines can be plotted to represent the eccentricity ratio, n versus the Sommerfeld number S, and we obtain a group of fan curves, on which the lines for l/d of 1, $^1/_2$, and $^1/_4$ can be labeled. On such a plot these lines are far enough apart for the l/d mentioned so that experimental data should show a trend quite readily. It is significant that our data for l/d of $^1/_4$, $^1/_2$, and 1 are in general agreement with the short-bearing approximation. The points show a moderate spread with the center slightly above the theoretical line. We expect that these data for the steady-load case will be available in an NACA technical note within a few months.

According to the short-bearing approximate method, we theoretically can plot the eccentricity ratio, n versus S $(l/d)^2$, making use of a new capacity number C_n, and obtain a single line. The experimental points of all of our tests again fall in a moderate spread slightly above this line. This capacity number, originated by our project, is mentioned by the author just above his Equation [12]. We believe that this capacity number may turn out to be of more practical value than the well-known Sommerfeld number for bearings of a short l/d ratio, up to 1 or a little more.

These theoretical curves are approximate in that they include the side flow which can be illustrated by a parabolic velocity profile, and they include the part of the circumferential flow which can be illustrated by a triangular velocity profile. However, they neglect the parabola superimposed on the triangle. The Sommerfeld solution, on the other hand, neglects the side flow and includes both parts of the circumferential flow. Thus it seems that the Sommerfeld theory and the short-bearing theory should be considered as the two halves of the same type of solution, rather than as being competitive. Each has a range of l/d in which it is the more useful. In our opinion, this short-bearing theory deserves wide recognition for its ability to include oil flow and give a useful approximation for short-bearing lengths commonly used.

The author also points out that some of the solutions assume the oil film for load purposes to have an extent of 360 deg, while others take 180 deg, neglecting the negative-pressure region. He also points out that the attitude angle is 90 deg for the 360-deg or 2π case, and approximates a semicircle for the 180-deg or π case. Our experimental data for unit loading up to 900 psi tend to follow the semicircle result.

In general, the experimental work reveals a number of practical considerations such as shaft deflection, variations in clearance, and in oil-inlet pressure which considerably affect the results, and tend to minimize the importance of minor error in the short-bearing approximation. This will be one of the most interesting phases of our report.

Referring to Fig. 9 (a) of the paper, the author states that at n = 0.8, the error in 1/S is about 20 per cent for an l/d of $^1/_4$, which is correct. If we use the same figure and enter with a value of S = $^1/_4$ or 1/S = 4, the two curves give values of n of about 79 per cent and 81 per cent—a difference of about 2 per cent.

On a plot of n versus S, the two theories in question give two lines close together, and the distance between them in the direction of the n-axis is small. On the other hand, intercepts in the direction of the S-axis are farther apart.

B. L. Newkirk.[3] Fig. 2 of the paper, with the theory pertaining to it, throws light on the question of stability of a journal in its equilibrium position. In the course of an experimental study of the oil-film whirl[4] it was found that an unloaded journal would whirl at a whirl frequency of one half its rotation frequency when running at low speed. At higher speed this whirl disappeared, but when the bearing was free to oscillate it did so, and at a frequency equal to one half the shaft rotation speed.

[1] By J. T. Burwell, published in the December, 1951, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 73, pp. 393–404.

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[4] "Shaft Whipping Due to Oil Action in Journal Bearings," *General Electric Review*, vol. 28, August, 1925, pp. 559–568.