Using the deflection function Equation [1] of this discussion, the Rayleigh-Ritz method yields a short formula for approximate value of the critical-load factor k_a

$$k_a = \frac{2b^2}{a^2} + \frac{12}{\pi^2}(1 - \nu) \qquad [3]^{11}$$

The simplicity of this formula for k_a, in comparison with the corresponding Equation [32] of the paper where the introduction of auxiliary functions requires almost a printed page, is striking indeed.

In Fig. 1 of this discussion k_a is plotted versus $\pi b/a$, for

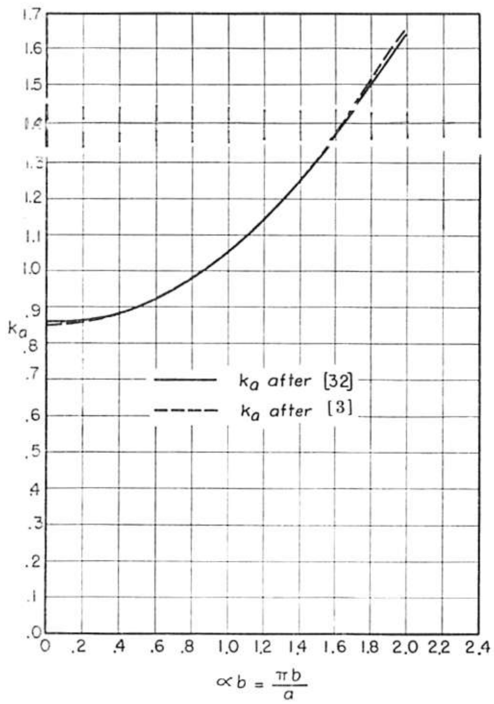

Fig. 1

$\nu = 0.30$, as obtained by Formula [32] of the paper, and taken from Fig. 3 of the paper (full curve), and as obtained by Equation [3], herewith, (dashed curve).

In the range where the exact buckling values have been computed ($0.5 < \pi b/a < 1.5$) the two formulas yield practically the same results. Since the Rayleigh-Ritz method gives always approximate eigenvalues which are too high, the simple formula, Equation [3] of this discussion, gives a better value in the range $0 < \pi b/a < 0.5$ than Equation [32]. In the range $1.5 < \pi b/a < 2.0$, Equation [3] is slightly worse than Equation [32] of the paper. Had an "admissible"[12] deflection function been chosen, involving more adjustable parameters than the single one (c) used in Equation [1] herewith, all the parameters should be determined by minimizing the energy expression, as required by the Rayleigh-Ritz procedure. If some of them would be determined by means of the natural boundary conditions, the resulting eigenvalue obviously would be larger and hence worse.

Author's Closure

The author wishes to express his thanks to Professors Bijlaard and Herrmann for their interest in his paper and for their helpful comments. In addition, he adds to these a brief summary of a letter written to the chairman of the session at which the paper was presented at the 1950 ASME Annual Meeting. The letter was from Dr. H. L. Engel of the Hughes Aircraft Company. He pointed out that an error was made in using bar theory when it was assumed that Γ was zero. The correct value of Γ is $^1/_{18}\, b^3t^3$ for the entire section, where b is the width of one flange and t is the thickness. With this correction it turns out that for short columns the agreement between bar and plate-theory calculations is very good, that is, within a few per cent.

[11] This is equation [99] in the first item of reference 10.

[12] One satisfying the artificial boundary conditions.

Transverse Vibrations of a Free Circular Plate Carrying Concentrated Mass[1]

J. E. Brock.[2] It is unfortunate that the author felt impelled to condense his paper to the extent he did. The writer had the opportunity of examining the analysis in an earlier and more extended stage and found the motivation much more evident there. While the first sentence of the section titled "Solution" tells what is done, the second contains either an error or such obscure notation that the reader comes to a full stop. In these two equations a symbol other than ξ should appear on the left side. The situation appears to be this: transforming Equations [4] leads to a differential equation for $W(\xi, s)$. This can be solved by transform methods; transforming with respect to ξ, one is led to examine a function $\overline{W}(x, s)$ and it is with the inversion of this function to obtain $W(\xi, s)$ that the formulas in question are concerned.

The use of impulse functions $\delta(r)$ seems to be a convenient way of accounting for the concentrated central mass and the fact that the exciting blow is struck at the center. The principal conclusions (natural frequencies and the significant observation that for small mass ratios these depend critically upon mass ratio) are, of course, independent of the fact that an impulsive loading is used.

Author's Closure

The difficulties at the beginning of the Solution to which Dr. Brock refers were due to some misprints in the preprint upon which his discussion was based. These misprints have been corrected in the paper as it appears in the Journal. It is still a matter of rather tedious manipulation to proceed from Equation [4] to [5]. The rationale of the procedure was felt to be obvious, but in any case is correctly inferred by Dr. Brock.

[1] By R. E. Roberson, published in the September, 1951, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 73, pp. 280–284.

[2] Director of Research, Midwest Piping and Supply Co., Inc., St. Louis, Mo. Mem. ASME.

On Elastic Continua With Hereditary Characteristics[1]

Karl Klotter.[2] The question of damping in materials already has been given considerable thought and has been attacked in various ways. The author in a previous paper[3] takes a new approach to this problem by linking it up in a rational way with the

[1] By Enrico Volterra, published in the September, 1951, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 73, pp. 273–279.

[2] Acting Associate Professor of Engineering Mechanics, Stanford University, Stanford, Calif.

[3] "Vibrations of Elastic Systems Having Hereditary Characteristics," by E. Volterra, Journal of Applied Mechanics, Trans. ASME, vol. 72, 1950, pp. 363–371.