



FIG. 1

ously been subjected to this stress. This condition is shown in Fig. 1, herewith, which illustrates approximate loading cycles at different sections of the rod. To a high degree of accuracy, uniform strain is obtained throughout the extent OB of the travel of the wave of maximum stress, and a stress, strain, strain-rate relation must be found which gives the same plastic strain for this widely different range of loading histories.

AUTHOR'S CLOSURE

The author wishes to thank Professors Wood and Lee for their pertinent discussion of the paper.

Perhaps the flow law should permit some instantaneous plastic response to applied stress, as suggested by Professor Wood; but the author doubts that a law of the particular form suggested would predict the region of constant strain near the impact end, since the factor $R(\dot{\epsilon})$ would not affect the maximum strain attained at different cross sections in the same degree because the strain-rate history varies greatly from one section to another near the impact end, as was pointed out by Professor Lee. It almost seems that the multiplier R should be a function of the velocity with which the impact end is struck instead of the strain rate in order to predict the constant-strain region, but such a dependence on velocity does not seem reasonable.

The author agrees with Professor Wood that a two-dimensional theory would be needed to describe properly the deformation in lead bars under compressive impact, when mushrooming occurs at the impact end. A law of the type used by the author, which predicts continued flow, could at best only account for the beginning of mushrooming.

Shakedown in Continuous Media¹

P. G. HODGE, JR.² Professor Symonds is certainly to be congratulated on having extended the concept of shakedown to the important case of continuous media. It is perhaps of some interest to notice that the inequality Equation [25] of the paper, obtained as a necessary condition for shakedown in a particular example, can, in fact, be generalized. To this end, let s_{ij}^* and s_{ij}^{**} be any two admissible elastic-stress states. Then, a necessary condition for shakedown is

$$\left(\frac{s_{ij}^{**} - s_{ij}^*}{2} \right) \left(\frac{s_{ij}^{**} - s_{ij}^*}{2} \right) \leq 2k^2 \dots \dots [1]$$

¹ By P. S. Symonds, published in the March, 1951, issue of the JOURNAL OF APPLIED MECHANICS, Trans. ASME, vol. 73, pp. 85-89.

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For assume that the Inequality [1] is false, then

$$s_{ij}^{\circ} s_{ij}^{\circ} > 8k^2 \dots \dots \dots [2]$$

where the notation $s_{ij}^{\circ} = s_{ij}^{**} - s_{ij}^*$ has been introduced. In addition, if there exists a shakedown state, it follows from the author's Equation [6] that

$$(\bar{r}_{ij} + s_{ij}^*) (\bar{r}_{ij} + s_{ij}^*) \leq 2k^2 \dots \dots \dots [3]$$

$$(\bar{r}_{ij} + s_{ij}^* + s_{ij}^{\circ}) (\bar{r}_{ij} + s_{ij}^* + s_{ij}^{\circ}) \leq 2k^2 \dots \dots [4]$$

However, it will now be shown that the Inequalities [2], [3], and [4], herewith, are inconsistent.

The Inequality [4] can be continued in the form

$$2k^2 \geq (\bar{r}_{ij} + s_{ij}^*) (\bar{r}_{ij} + s_{ij}^*) + 2 s_{ij}^{\circ} (\bar{r}_{ij} + s_{ij}^*) + s_{ij}^{\circ} s_{ij}^{\circ} = (\bar{r}_{ij} + s_{ij}^*) (\bar{r}_{ij} + s_{ij}^*) - 2 |s_{ij}^{\circ} (\bar{r}_{ij} + s_{ij}^*)| + s_{ij}^{\circ} s_{ij}^{\circ} \dots \dots \dots [5]$$

The second step is valid, since in view of Inequality [2] of this discussion and the fact that the first term in the middle member of Equation [5] is nonnegative, the second term is less than zero. In view of Cauchy's inequality (following author's Equation [16]), Equation [5] leads to

$$2k^2 \geq (\bar{r}_{ij} + s_{ij}^*) (\bar{r}_{ij} + s_{ij}^*) - 2 [(\bar{r}_{ij} + s_{ij}^*) (\bar{r}_{ij} + s_{ij}^*) \times s_{mn}^{\circ} s_{mn}^{\circ}]^{1/2} + s_{mn}^{\circ} s_{mn}^{\circ} = [\sqrt{s_{mn}^{\circ} s_{mn}^{\circ}} - \sqrt{(\bar{r}_{ij} + s_{ij}^*) (\bar{r}_{ij} + s_{ij}^*)}]^2 \dots \dots \dots [6]$$

Finally, it follows from Expressions [6], [2], and [3], herewith, that

$$2k^2 > [\sqrt{8k^2} - \sqrt{2k^2}]^2 = 2k^2 \dots \dots \dots [7]$$

which is an obvious contradiction. Thus the Inequality [1] is a necessary (although, in general, not a sufficient) condition for shakedown.

AUTHOR'S CLOSURE

Professor Hodge's contribution is appreciated. Unfortunately, as he states, the theorem he proves gives a necessary but not sufficient condition for shakedown. An interesting subject for future study is the relation between the theorems concerning the load limits for which shakedown just occurs, and those which recently have been proved concerning the loads at which unrestricted plastic flow occurs under steady proportional loading of an elastic perfectly plastic body.^{3,4} In the case of continuous frame structures close analogies exist between the analytical methods for proportional loading and for variable, repeated loads.^{5,6} It would be of great interest to find whether similar analogies can be utilized also in the case of continuous media.

³ "The Safety Factor of an Elastic-Plastic Body in Plane Strain," by D. C. Drucker, H. J. Greenberg, and W. Prager, Paper No. 51-A-3, to be presented at the Annual Meeting, Atlantic City, N. J., Nov. 25-30, 1951, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS.

⁴ "Extended Limit Theorems for Structures and Continuous Media," by D. C. Drucker, W. Prager, and H. J. Greenberg, technical report, No. 59, Brown University to Office of Naval Research, March, 1951.

⁵ "A Method for Calculating the Failure Load for a Framed Structure Subjected to Fluctuating Loads," by B. G. Neal and P. S. Symonds, *Journal of the Institution of Civil Engineers* (London), vol. 35, 1950-1951, pp. 186-198.

⁶ "New Techniques for Computing Plastic Failure Loads of Continuous Frame Structures," by P. S. Symonds and B. G. Neal, paper presented at First U. S. National Congress of Applied Mechanics, Chicago, Ill., June 11-16, 1951, technical report No. 62, Brown University to Office of Naval Research, August, 1951.