and temperature. Such a metal, which must retain some of the original void volume of the sphere bed to remain permeable, is typified by the commercial porous-bronze specimen shown in the authors' Fig. 1. It may be seen from this figure that the identity of the individual grains is largely preserved, and in this case knowledge of the grain diameter (and possibly of a suitable shape factor) may permit a prediction of the flow resistance of the compact by the standard methods of fluid mechanics.

In the method of manufacture used by the authors, on the other hand, the metal powder is mixed with a powdered solid which undergoes a phase change into a gaseous state at a temperature slightly below the sintering temperature of the metal. This intimate mixture is then pressed in a die under pressures ranging up to 100,000 psi, and the resulting compact sintered at high temperature. The permeability of the metal is thus due to the pores formed by the escape of the gases generated by the decomposition of the porosity-forming agent. A metal prepared by this technique is typified by the stainless-steel specimen shown in the authors' Fig. 1, from which the loss of particle identity may be seen. This loss of identity, although it complicates the flow picture, is rewarded by a mechanical strength in the compact greater than that obtained by the conventional technique.

In closing, the authors would like to express their preference for the granular bed correlation of Ergun and Orning[7] over that of Rose,[4] since they believe that the two-term quadratic expression of Ergun and Orning gives a clearer picture of the resistance mechanism. In addition, the dimensionless coefficient of the quadratic term (see the authors' Equation [16]) provides a measure of the effect of particle orientation (the mode of sphere packing) upon the flow resistance. This effect has been neglected by Rose, who considers the resistance of a sphere bed (in the absence of wall effects) to be completely described by the particle diameter and bed porosity, but recent work by Martin, McCabe, and Monrad[8] has shown it to be important in certain cases of systematic sphere packing.

The Solution of Elastic Plate Problems by Electrical Analogies[1]

M. V. Barton.[2] The use of an analog computer for the solution of plate problems is valuable for the study of complicated plates having difficult boundary conditions, or in which the plate is variable in thickness, or has unusual plan form. Since, however, the differential equations to be solved for the plate have been approximated in terms of finite differences, the function of the computer is to solve sets of simultaneous algebraic equations representing particular boundary value or characteristic-value problems. Therefore it may be of interest to compare the results obtained by the analog computer with results obtained by brute-force calculations.

It has been the experience of the writer that the type of characteristic-value problem such as the vibration of the rectangular plate discussed in the paper is very laborious to solve by usual numerical procedures. For example, using an iteration procedure on a set of 12 finite-difference equations (same number as used by the author), the characteristic value continues to oscillate between plus and minus values after 24 iterations. An alternative method is to expand the determinant of the equations to obtain the characteristic equation, which in this case is a twelve-degree polynominal, and extract the roots of interest. This has been done to determine the lowest characteristic value for the vibrating cantilever plate. The frequency is found to be $0.503\ 1/a^2\sqrt{D/m}$, as compared with the author's value of $0.501\ 1/a^2\sqrt{D/m}$. The close correlation of these values is an indication of the accuracy of the computer. As the author points out, these values are much lower than the test frequency because of the approximation inherent in the finite-difference method so that comparisons with experimental results are not a good basis for judging the computer's effectiveness.

It is interesting to note that the values of frequency obtained for the vibrating cantilever plate using the twelve equations are lower bounds. The value of the fundamental frequency obtained by the Ritz method using a 9-term series of orthogonal beam functions to represent the deflection gives an upper bound of $0.556\ 1/a^2\sqrt{D/m}$ which is less than 2 per cent higher than the experimental value. Therefore it may be desirable to use the analog computer to solve the set of equations resulting from the application of the Ritz method rather than using the finite-difference method since for some problems greater accuracy can be obtained with fewer cells.

P. G. Hodge, Jr.[3] The author's solution of the elastic plate problem was particularly interesting to the writer because it can be applied immediately to the elastic plane-strain problem. As is well known, if a stress function ψ is defined by

$$\sigma_x = \partial^2\psi/\partial_y^2,\ \sigma_y = \partial^2\psi/\partial_x^2,\ \tau_{xy} = -\partial^2\psi/\partial_x\partial_y$$

then ψ must satisfy the author's Equation [10] with the right-hand side equal to zero

$$\frac{\partial^4\psi}{\partial_x^4} + 2\frac{\partial^4\psi}{\partial_x^2\partial_y^2} + \frac{\partial^4\psi}{\partial_y^4} = 0 \quad\quad [1]$$

If the region under consideration is simply connected, the boundary conditions for ψ can be expressed in terms of the stress vector $\mathbf{T}$ applied to the boundary of the region in plane strain. If $\mathbf{n}$ and $\mathbf{t}$ are unit vectors, respectively, normal and tangential to the boundary

$$\psi = \int_0^s \int_0^s \mathbf{T}\cdot\mathbf{n}\, ds\, ds,\ \frac{\partial\psi}{\partial n} = -\int_0^s \mathbf{T}\cdot\mathbf{t}\, ds \quad\quad [2]$$

where the integrations are to be taken around the boundary. Thus the boundary conditions can always be computed in the form of Equations [a] and [c] of the author's paper. It follows, then, that exactly the same network can be used to solve a problem in plane elastic strain.

The writer would now like to pose the much more difficult problem of elastic-plastic plane strain. Here the stress function ψ must be continuous with continuous first and second derivatives throughout the region, must satisfy the boundary conditions, Equation [2] of this discussion, and the yield inequality

$$\left[\frac{\partial^2\psi}{\partial_x^2} - \frac{\partial^2\psi}{\partial_y^2}\right]^2 + 4\left[\frac{\partial^2\psi}{\partial_x\partial_y}\right]^2 - 4k^2 \leq 0 \quad\quad [3]$$

[7] "Fluid Flow Through Randomly Packed Columns and Fluidized Beds," by S. Ergun and A. A. Orning, *Industrial and Engineering Chemistry*, vol. 41, 1949, p. 1179.

[8] "Pressure Drop Through Stacked Spheres: Effect of Orientation," by J. J. Martin, W. L. McCabe, and C. C. Monrad, *Chemical Engineering Progress*, vol. 47, 1951, p. 91.

[1] By R. H. MacNeal, published in the March, 1951, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 73, pp. 59–67.

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Wherever the inequality in Equation [3] herewith is valid, it must satisfy Equation [1]. This problem is an exceedingly difficult one analytically, and only a very few special cases have been solved.[4] Therefore, if it were possible to construct an electrical analog for this case, it would be a valuable contribution to elastic-plastic theory. This would involve a nonlinear element, which would, in effect, put a "ceiling" on the current that could be carried by any element. Current which would violate this ceiling must then be "rerouted" via other elements in such a way as to preserve the continuity conditions.

Author's Closure

The author wishes to thank Dr. Barton for the numerical verification of the analog-computer results. In the author's opinion, the reason for the lack of agreement between the experimental value for the lowest mode frequencies and the values calculated by finite differences lies in a crude and fortunately avoidable approximation in the distribution of inertias. In the author's paper and presumably in Dr. Barton's calculations, unit masses are connected to interior points; $^1/_2$ unit masses are connected to edge points and $^1/_4$ unit masses are connected to corner points. If instead a distribution is adopted that is statically equivalent to the distributed mass load, better results will be obtained. The square cantilever plate problem has recently been repeated using a finer-mesh (5 cells on a side). The computed frequency is $0.542 \frac{1}{a^2} \sqrt{D/m}$ which compares well with Dr. Barton's experimental result which is $0.546 \frac{1}{a^2} \sqrt{D/m}$.

It is probably not practical to use the analog computer to solve the set of equations resulting from an application of the Ritz method. In any event the Ritz method does not produce good values for stresses while in the finite difference method the calculated stresses satisfy statics exactly.

The author wishes to point out to Professor Hodge that the problem of constructing an electric analogy for the elastic plane-strain problem was investigated by G. Kron[5] a few years ago. His plane-strain network was derived as a special case of a network representing a three-dimensional elastic field. Network analyzer solutions were presented in a companion paper.[6]

One form of the plane-strain circuit is similar to the network shown in the author's paper with the W-circuit removed.

The construction of an analog for the elastic-plastic plane-strain problem would be difficult. If the yield inequality were imposed directly on the stress components

$$|\sigma_x| < K_1 \qquad |\sigma_y| < K_2 \qquad |\tau_{xy}| < K_3$$

these conditions could be handled by means of current limiters employing crystal diodes. Unfortunately the condition is imposed on the principal stress as shown in the foregoing Equation [3], which condition cannot easily be imposed on an electric circuit.

[4] "Plane Elastic-Plastic Problem: Plastic Regions Around Circular Holes in Plates and Beams (Russian)," by L. A. Galin, *Prikladnaia Matematika i Mekhanika*, vol. 10, 1946, pp. 365–386; "An Elastic-Plastic Problem With a Nonbiharmonic Plastic State (Russian)," by O. S. Parasyuk, ibid., vol. 13, 1948, pp. 367–370.

[5] "Equivalent Circuits of the Elastic Field," by G. Kron, Journal of Applied Mechanics, Trans. ASME, vol. 66, 1944, pp. A-149–161.

[6] "Numerical and Network Analyzer Solution of the Equivalent Circuits for the Elastic Field" by G. Carter, Journal of Applied Mechanics, Trans. ASME, vol. 66, 1944, pp. A-162–167.

The Propagation of Longitudinal Waves of Plastic Deformation in a Bar of Material Exhibiting a Strain-Rate Effect[1]

D. S. Wood.[2] The author has assumed a certain flow law in which the stress depends upon strain rate as well as strain, in an attempt to obtain a better agreement between the theory of plastic strain waves and experimental observations of these waves. This is a good example of the type of procedure which must be followed in order to determine the mechanical properties of materials at the very high strain rates reached in impact tests. The particular flow law assumed by the author leads to improved agreement between theory and experiment in one respect, but, unfortunately, one of the most prominent features of the experimental results is lost, namely, the region of constant plastic strain near the impact end. The writer has the feeling that this disagreement is associated with the fact that the assumed flow law requires purely elastic response of the material to instantaneous finite changes of stress. This implies that the correct flow law must be such that instantaneous finite changes of stress are, in general, accompanied by instantaneous finite changes of plastic strain. A flow law of this type is

$$\sigma = h(\epsilon) - R(\dot{\epsilon})[h(\epsilon) - f(\epsilon)]$$

where

$f(\epsilon)$ represents the static stress-strain relation
$h(\epsilon)$ represents a stress-strain relation for instantaneously applied stresses
$R(\dot{\epsilon})$ is a function of the strain rate such that
$R(0) = 1$, and $R(\infty) = 0$

With regard to the results of compressive impact tests on lead (20), referred to by the author, it should be noted that those specimens which did not show a region of constant plastic strain near the impact end exhibited mushrooming at the impact end; that is, marked lateral flow occurred at the impact surface. For this reason, it appears that any theory properly describing these particular experimental results must be a two-dimensional rather than a one-dimensional theory. Hence, it is difficult to see how a theory of the type presented by the author can describe these results properly.

E. H. Lee.[3] The numerical integration, the results of which are given in this paper, illustrates the length and complication of a wave analysis for a stress-strain relation involving a strain-rate dependence. Although the assumed, seemingly reasonable, relationship does not reproduce all the experimental results, it is valuable in emphasizing features of the stress, strain, strain-rate relation required.

The remarkable requirements called for by the experimental results cited in references (18) and (19) of the paper, have not, it is believed, been generally appreciated. After impact a region of uniform permanent strain is obtained, which comprises material which has been subjected to the maximum stress throughout the duration of impact, and material which has only instantane-

[1] By L. E. Malvern, published in the June, 1951, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 73, pp. 203–208.

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