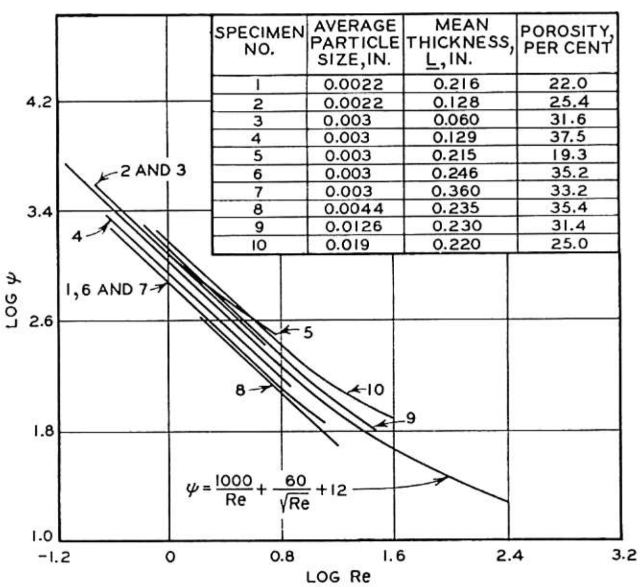
Fluid Flow Through Porous Metals[1]

P. Grootenhuis.[2] A knowledge of the fluid flow through porous materials made by powder-metallurgy methods is becoming of importance with the advent of sweat cooling. The authors have made a valuable contribution, and the method of correlating the experimental data is most interesting. It must be remembered, however, that such a correlation depends upon the determination of the two empirical constants α and β as defined in Equation [4] or [11] of the paper. An experimental determination of the pressure drop-flow characteristic must be made to evaluate these constants for each different type of porous compact. Such an experiment might even be necessary for compacts made of powders of substantially the same particle shape but of varying particle size.

One of the most important objects of a correlation of experimental data as presented in the paper, Fig. 4, should be that no further experiments are necessary to determine similar data for new specimens. It seems that the correlation falls short of this requirement.

The results of similar experiments on some porous metallic compacts were published by the present writer in 1949.[3] The pressure-drop versus flow characteristics were determined for a number of porous-bronze disks, of various thicknesses, and made from powders substantially spherical in shape but varying in size. The results were correlated in a manner analogous to that suggested by Rose[4] for fluid flow through beds of granular materials. The correlation of some of the results is given in Fig. 1, herewith. The resistance coefficient ψ for the isothermal flow of gases is defined as

$$\psi = \frac{(\rho_1^2 - \rho_2^2)\, g\, d}{2\, C\, T\, G^2\, L\, F(f)}$$



SPECIMEN NO.	AVERAGE PARTICLE SIZE, IN.	MEAN THICKNESS, L, IN.	POROSITY, PER CENT
1	0.0022	0.216	22.0
2	0.0022	0.128	25.4
3	0.003	0.060	31.6
4	0.003	0.129	37.5
5	0.003	0.215	19.3
6	0.003	0.246	35.2
7	0.003	0.360	33.2
8	0.0044	0.235	35.4
9	0.0126	0.230	31.4
10	0.019	0.220	25.0

Fig. 1 Correlation of Results for Fluid Flow Through Beds of Granular Materials

where

ρ_1 = upstream pressure
ρ_2 = downstream pressure
d = average particle size
C = gas constant
T = absolute temperature
G = weight rate of flow per unit area
L = thickness of the specimen

and $F(f)$ is a function of the porosity, f

$$= \frac{2.4\,(1 - f)^{3.5}}{f}$$

The second characteristic used is the Reynolds number $Re = Gd/\mu$, where μ is the absolute viscosity of the gas.

This correlation depends upon only one empirical relationship, namely, the porosity function $F(f)$. The numerical constants of this function have been derived from the numerous experiments on beds of granular materials of widely varying properties. The graph shows that the correlation between the flow data for the porous disks and the generalized curve by Rose is reasonably good. No allowance has been made for the inevitable distortion of the particles during manufacture. The greatest divergence is shown by specimens 8 and 10, which were made from powders having a poor particle-size distribution.

This correlation shows that as a first approximation, these porous metal disks have flow characteristics similar to those of beds of packed granulars and that, by making use of the well-accepted correlation formulas for such granular materials, the flow properties of new porous metals can be estimated without having to resort to experiments. It is only necessary to measure the average particle size of the powder used in the manufacture. This usually has to be known in any case, to determine the pressing and sintering conditions needed to make the required compact.

It would be of interest if the authors could estimate the average particle size of the powders used and recalculate their experimental data as suggested in these notes. For nonspherical particles the porosity function and shape factors as given by Rose and Rizk[5] or by Coulson[6] should be used.

Authors' Closure

The authors wish to thank Mr. Grootenhuis for his interesting discussion. His suggestion that the porous-metal flow data presented by the authors be correlated in terms of the particle size of the powder forming the compact, however, indicates that the authors have not sufficiently emphasized the raison d'être of their paper. This fact is that the loss of identity suffered by the individual particles in the method of compact manufacture employed by the authors is so complete that no analytical correlation between the original particle size and the flow resistance of the compact has been found to exist. Such a situation required the use of two empirically determined characteristic-length parameters, such as the coefficients α and β defined by the authors.

The authors were careful to point out that in the ideal case of a bed of spherical particles, flow data could be correlated with the use of a single length parameter, the particle diameter. Such a correlation is of course possible with porous metals prepared by the conventional method presumably used by Mr. Grootenhuis, in which closely sized spherical particles, compacted lightly if at all, are sintered under closely controlled conditions of time

[1] By L. Green, Jr., and P. Duwez, published in the March, 1951, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 73, p. 39.

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[3] "The Flow of Gases Through Porous Metal Compacts," by P. Grootenhuis, *Engineering*, vol. 167, April 1, 1949, p. 291.

[4] "An Investigation Into the Laws of Flow of Fluids Through Beds of Granular Materials," by H. E. Rose, Proceedings of The Institution of Mechanical Engineers, vol. 153, 1945, p. 141.

[5] "Further Researches in Fluid Flow Through Beds of Granular Material," by H. E. Rose and A.M.A. Rizk, Proceedings of The Institution of Mechanical Engineers, vol. 160, 1949, p. 493.

[6] "The Flow of Fluids Through Granular Beds," by J. M. Coulson, Institution of Chemical Engineers, vol. 27, December, 1949.

and temperature. Such a metal, which must retain some of the original void volume of the sphere bed to remain permeable, is typified by the commercial porous-bronze specimen shown in the authors' Fig. 1. It may be seen from this figure that the identity of the individual grains is largely preserved, and in this case knowledge of the grain diameter (and possibly of a suitable shape factor) may permit a prediction of the flow resistance of the compact by the standard methods of fluid mechanics.

In the method of manufacture used by the authors, on the other hand, the metal powder is mixed with a powdered solid which undergoes a phase change into a gaseous state at a temperature slightly below the sintering temperature of the metal. This intimate mixture is then pressed in a die under pressures ranging up to 100,000 psi, and the resulting compact sintered at high temperature. The permeability of the metal is thus due to the pores formed by the escape of the gases generated by the decomposition of the porosity-forming agent. A metal prepared by this technique is typified by the stainless-steel specimen shown in the authors' Fig. 1, from which the loss of particle identity may be seen. This loss of identity, although it complicates the flow picture, is rewarded by a mechanical strength in the compact greater than that obtained by the conventional technique.

In closing, the authors would like to express their preference for the granular bed correlation of Ergun and Orning[7] over that of Rose,[4] since they believe that the two-term quadratic expression of Ergun and Orning gives a clearer picture of the resistance mechanism. In addition, the dimensionless coefficient of the quadratic term (see the authors' Equation [16]) provides a measure of the effect of particle orientation (the mode of sphere packing) upon the flow resistance. This effect has been neglected by Rose, who considers the resistance of a sphere bed (in the absence of wall effects) to be completely described by the particle diameter and bed porosity, but recent work by Martin, McCabe, and Monrad[8] has shown it to be important in certain cases of systematic sphere packing.

The Solution of Elastic Plate Problems by Electrical Analogies[1]

M. V. Barton.[2] The use of an analog computer for the solution of plate problems is valuable for the study of complicated plates having difficult boundary conditions, or in which the plate is variable in thickness, or has unusual plan form. Since, however, the differential equations to be solved for the plate have been approximated in terms of finite differences, the function of the computer is to solve sets of simultaneous algebraic equations representing particular boundary value or characteristic-value problems. Therefore it may be of interest to compare the results obtained by the analog computer with results obtained by brute-force calculations.

It has been the experience of the writer that the type of characteristic-value problem such as the vibration of the rectangular plate discussed in the paper is very laborious to solve by usual numerical procedures. For example, using an iteration procedure on a set of 12 finite-difference equations (same number as used by the author), the characteristic value continues to oscillate between plus and minus values after 24 iterations. An alternative method is to expand the determinant of the equations to obtain the characteristic equation, which in this case is a twelve-degree polynominal, and extract the roots of interest. This has been done to determine the lowest characteristic value for the vibrating cantilever plate. The frequency is found to be 0.503 $1/a^2\sqrt{D/m}$, as compared with the author's value of 0.501 $1/a^2 \sqrt{D/m}$. The close correlation of these values is an indication of the accuracy of the computer. As the author points out, these values are much lower than the test frequency because of the approximation inherent in the finite-difference method so that comparisons with experimental results are not a good basis for judging the computer's effectiveness.

It is interesting to note that the values of frequency obtained for the vibrating cantilever plate using the twelve equations are lower bounds. The value of the fundamental frequency obtained by the Ritz method using a 9-term series of orthogonal beam functions to represent the deflection gives an upper bound of 0.556 $1/a^2\sqrt{D/m}$ which is less than 2 per cent higher than the experimental value. Therefore it may be desirable to use the analog computer to solve the set of equations resulting from the application of the Ritz method rather than using the finite-difference method since for some problems greater accuracy can be obtained with fewer cells.

P. G. Hodge, Jr.[3] The author's solution of the elastic plate problem was particularly interesting to the writer because it can be applied immediately to the elastic plane-strain problem. As is well known, if a stress function ψ is defined by

$$\sigma_x = \partial^2\psi/\partial_y{}^2,\ \sigma_y = \partial^2\psi/\partial_x{}^2,\ \tau_{xy} = -\partial^2\psi/\partial_x\partial_y$$

then ψ must satisfy the author's Equation [10] with the right-hand side equal to zero

$$\frac{\partial^4\psi}{\partial_x{}^4} + 2\frac{\partial^4\psi}{\partial_x{}^2\partial_y{}^2} + \frac{\partial^4\psi}{\partial_y{}^4} = 0 \qquad [1]$$

If the region under consideration is simply connected, the boundary conditions for ψ can be expressed in terms of the stress vector $\mathbf{T}$ applied to the boundary of the region in plane strain. If $\mathbf{n}$ and $\mathbf{t}$ are unit vectors, respectively, normal and tangential to the boundary

$$\psi = \int_0^s\int_0^s \mathbf{T}\cdot\mathbf{n}\, ds\, ds,\ \frac{\partial\psi}{\partial n} = -\int_0^s \mathbf{T}\cdot\mathbf{t}\, ds \qquad [2]$$

where the integrations are to be taken around the boundary. Thus the boundary conditions can always be computed in the form of Equations [a] and [c] of the author's paper. It follows, then, that exactly the same network can be used to solve a problem in plane elastic strain.

The writer would now like to pose the much more difficult problem of elastic-plastic plane strain. Here the stress function ψ must be continuous with continuous first and second derivatives throughout the region, must satisfy the boundary conditions, Equation [2] of this discussion, and the yield inequality

$$\left[\frac{\partial^2\psi}{\partial_x{}^2} - \frac{\partial^2\psi}{\partial_y{}^2}\right]^2 + 4\left[\frac{\partial^2\psi}{\partial_x\partial_y}\right]^2 - 4k^2 \leq 0 \qquad [3]$$

[7] "Fluid Flow Through Randomly Packed Columns and Fluidized Beds," by S. Ergun and A. A. Orning, *Industrial and Engineering Chemistry*, vol. 41, 1949, p. 1179.

[8] "Pressure Drop Through Stacked Spheres: Effect of Orientation," by J. J. Martin, W. L. McCabe, and C. C. Monrad, *Chemical Engineering Progress*, vol. 47, 1951, p. 91.

[1] By R. H. MacNeal, published in the March, 1951, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 73, pp. 59–67.

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