

# Discussion

## Evaluation of Stress Distribution in the Symmetrical Neck of Flat Tensile Bars<sup>1</sup>

E. H. LEE.<sup>2</sup> Presumably the type of strain distribution considered by the author applies only for a range of width-to-thickness ratios, for with flat wide specimens the constraint of the width prevents contraction, and necking occurs through the thickness of the specimen. Thickness gradients then become large, and the system no longer can be considered as in plane stress. It seems that there is a range of width-thickness ratios sufficiently large to permit the assumption of plane stress, and sufficiently small to avoid sharp necks producing local reduction in the thickness. Perhaps the author would comment on the range of validity of this type of behavior.

### AUTHOR'S CLOSURE

The comments made by Professor Lee are appreciated. The author agrees, as was stated in the text, that the type of strain distribution considered can be applied only for a relatively narrow-range width-to-thickness ratio.

Presumably the type of sharp necks in the thickness direction that Professor Lee described for larger width-to-thickness ratios is the same type of necking described in the literature by Koerber, Siebel, Bijlaard, and Hill.

A separate investigation was carried out by the author to study what effect the width-to-thickness ratio and strain-hardening has on the mode of necking. These results were presented at the First U. S. National Congress of Applied Mechanics, June, 1951, at Chicago, Ill. The later results indicate that the mode of necking will undergo a transition from the symmetrical-type neck to the oblique neck as the width-to-thickness ratio is increased. However, there are not sufficient data available to make a general statement.

## Free Vibrations of a Pin-Ended Column With Constant Distance Between Pin Ends<sup>1</sup>

N. J. HOFF.<sup>2</sup> The most important result obtained in the paper is an explanation of the failure of attempts to establish a nondestructive method of testing the elastic stability of structures. It often has been proposed that such structures should be loaded up to as high values of the loads as can be reached without danger of permanent deformation. At different load levels the lowest natural mode of vibration of the structure should be excited. The curve representing the natural frequencies plotted against the

<sup>1</sup> By Julius Aronofsky, published in the March, 1951, issue of the *JOURNAL OF APPLIED MECHANICS*, Trans. ASME, vol. 13, pp. 75-84.

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<sup>1</sup> By David Burgreen, published in the June, 1951, issue of the *JOURNAL OF APPLIED MECHANICS*, Trans. ASME, vol. 73, pp. 135-139.

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load can be extrapolated, and the buckling load of the structure is the load at which the natural frequency is zero.

The author has shown that in an ordinary column test the natural frequency depends greatly upon the amplitude of the vibration, and the frequency is considerably higher than zero at the instability load. As an example, it might be mentioned that when the amplitude is one half the radius of gyration of the cross section, the frequency at the buckling load is equal to 20 per cent of the natural frequency of a column upon which no end load is acting. This is true, naturally, only if the load is applied by means of controlled end displacements, as is done in an ordinary testing machine, and not through the application of dead weights. Consequently, extrapolation of the curve to zero frequency results in a load differing substantially from the critical load of the classical theory of stability. Moreover, variations in the amplitude of the vibrations at different stages of the loading cause a great deal of scatter in the results and add to the difficulties of obtaining the buckling load by means of vibration tests.

It appears, therefore, that attempts at determining the buckling load by such nondestructive tests either have to be given up or else will have to be modified in the light of the author's results.

E. H. LEE.<sup>3</sup> With reference to the author's comments on the restriction on the initial shape of the strut, it may be worth mentioning that this type of analysis will go through for an initial eccentricity comprising a sine curve with any integral number of half waves. While this provides an infinite number of initial configurations, they cannot be superimposed to cover any initial shape since the problem is nonlinear.

## Vibration of Rectangular and Skew Cantilever Plates<sup>1</sup>

M. Z. v. KRZYWOBLOCKI.<sup>2</sup> As mentioned in the paper, the Rayleigh-Ritz method gives an upper bound, but it is possible to find a lower bound. The combination: The Rayleigh-Ritz and the Weinstein methods proved to be a successful mathematical tool, giving upper and lower bounds. It may be of interest to apply this combination in the case considered by the author.

## Analysis of Deep Beams<sup>1</sup>

A. J. DURELLI.<sup>2</sup> The problem dealt with here by the authors is very important in many industrial applications, and the contribution made by them will certainly be welcome since the study is far from exhausted.

It may be worth mentioning that a series of papers was pub-

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<sup>1</sup> By M. V. Barton, published in the June, 1951, issue of the *JOURNAL OF APPLIED MECHANICS*, Trans. ASME, vol. 73, pp. 129-134.

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<sup>1</sup> By H. D. Conway, L. Chow, and G. W. Morgan, published in the June, 1951, issue of *JOURNAL OF APPLIED MECHANICS*, Trans. ASME, vol. 73, pp. 163-172.

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