creases; for instance, for $P/P_E = 0.5$ this value is approximately 2.5, and for $P/P_E = 0.9$ it is approximately 2.

Curves giving maximum-moment coefficients β and γ, using results from the previous study,[3] are shown in Figs. 1 and 2 herewith. These figures should replace the author's Figs. 9 and 10. For comparison, the moment coefficients for the center of the plate are shown in dotted lines for those regions where they are not maximum.

The range of aspect ratios $a/b < 1$ was not investigated, but, under certain conditions, maximum moments may be expected to occur, not at the center of the plate, but at some other point on the center line parallel to the y-axis. Some indication of what happens can be obtained from the solution for the limiting case $a/b = 0$. This is easily derived by introducing a new co-ordinate $z = y + (b/2)$, representing the distance from the edge $y = -b/2$. Substituting for y its value in terms of z, and for A_m and C_m their values, Equations [19], into Equation [18] of the paper and then letting b approach infinity, the following equation for the deflection is obtained

$$\omega = \frac{4q}{\pi} \sum_{m=1,3,5\ldots}^{\infty} \frac{\eta_m{}^2 e^{-\gamma_m z} - \gamma_m{}^2 e^{-\eta_m z} + 2\alpha_m \sqrt{\dfrac{P}{D}}}{2m\alpha_m{}^3 \sqrt{PD}\left(\alpha_m{}^2 - \dfrac{P}{D}\right)} \sin \frac{m\pi x}{a}$$

The following equations can then be written for the derivatives of the bending moments along the center line parallel to the y-axis

$$\left[\frac{\partial M_x}{\partial z}\right]_{x=a/2} = \sum_{m=1,3,5\ldots}^{\infty} (-1)^{\frac{m-1}{2}} R_m \left[(\alpha_m{}^2 - \nu\eta_m{}^2)\gamma_m e^{(\gamma_m - \eta_m)z} - (\alpha_m{}^2 - \nu\gamma_m{}^2)\eta_m\right]$$

$$\left[\frac{\partial M_y}{\partial z}\right]_{x=a/2} = \sum_{m=1,3,5\ldots}^{\infty} (-1)^{\frac{m-1}{2}} R_m \left[(\nu\alpha_m{}^2 - \eta_m{}^2)\gamma_m e^{(\gamma_m - \eta_m)z} - (\nu\alpha_m{}^2 - \gamma_m{}^2)\eta_m\right]$$

where

$$R_m = \frac{2q\gamma_m\eta_m e^{-\gamma_m z}}{m\pi\alpha_m{}^3\sqrt{P/D}\left(\alpha_m{}^2 - \dfrac{P}{D}\right)}$$

Since an infinitely wide plate buckles essentially as a column, the following inequalities apply for any m

$$\alpha_m{}^2 > P/D$$
$$\gamma_m > \alpha_m > \eta_m > 0$$
$$R_m > 0$$

In investigating the signs of the derivatives of the bending moments, it will be sufficient to consider only the first term of each series, as these converge very rapidly. With this approximation, for $z = 0$

$$\left[\frac{\partial M_x}{\partial z}\right]_{x=a/2;\ z=0} \cong R_1(\gamma_1 - \eta_1)(\alpha_1{}^2 + \nu\gamma_1\eta_1) > 0$$

$$\left[\frac{\partial M_y}{\partial z}\right]_{x=a/2;\ z=0} \cong R_1(\gamma_1 - \eta_1)(\nu\alpha_1{}^2 + \gamma_1\eta_1) > 0$$

For the moments M_x and M_y to have a maximum at a finite distance from the edge $z = 0$, their derivatives would have to become negative for some value of z. Since $e^{(\gamma_1 - \eta_1)z}$ can have any value between $+1$ and $+\infty$ as z varies, this will only happen, for M_x and M_y, respectively, if $(\alpha^2{}_1 - \nu\eta_1{}^2) < 0$ and $(\nu\alpha_1{}^2 - \eta_1{}^2) < 0$. From the inequalities just set forth and from the expression for η_1 in terms of α_1, it follows that M_x cannot have a maximum at a finite distance from the edge while M_y will if

$$P < \frac{\pi^2 D}{a^2}(1 - \nu)^2$$

From the foregoing discussion, the following conclusions—which supplement and modify some of the author's—can be arrived at for the case of compressive end load:

1 Throughout the whole range of aspect ratios from zero to infinity, maximum bending moments are never independent of P. As Figs 1 and 2 herewith show, these moments are always larger than the corresponding maximum moments for the case $P = 0$.

2 For very long plates ($a/b >> 1$), maximum bending moments cannot be calculated by means of cylindrical-surface formulas, except for M_y in the limiting case $P = 0$.

3 For very short plates ($a/b << 1$), it appears that the maximum moment M_x can be calculated by means of cylindrical-surface formulas for all values of P. On the other hand, such formulas will be applicable to the calculation of the maximum moment M_y only if $P > (\pi^2 D/a^2)(1 - \nu)^2$.

Author's Closure

The author is indebted to Mr. Contini for his discussion. For the sake of mathematical elegance it is probably desirable to sum parts of Equations [14] and [21]. However, in the example given by Mr. Contini, the first two terms of the author's series give a result which is only 1 per cent in error; such convergence is quite satisfactory for any practical problem.

The fact that, for certain values of a/b and end load, the maximum bending moments will not occur at the center of the plate is interesting and subscripts "max" in the paper should therefore be changed to "center." However, the author considers this to be of academic interest only and the maximum resultant stress in plates with practical values of a/b and compression end load can usually be calculated as occurring at the center.

The Dynamics of Cavitation Bubbles[1]

Edward Silberman.[2] The writer has made some qualitative observations regarding the initial stages of bubble formation in low-pressure regions which may be of interest to readers of this paper. The observations were made incidental to experiments on resorption of air bubbles in turbulent water.[3] The experimental apparatus consisted essentially of a sealed lucite cylinder, filled with distilled water. A brass rotor was installed in the cylinder near one end and connected by a shaft to an external motor. The liquid inside the cylinder could be made to rotate in a turbulent state at various speeds by controlling the speed of the motor. A pressure connection to the cylinder (through a sealed diaphragm) permitted wide variation in superimposed pressure. Observation was with the unaided eye under a strong continuous light, or with an 18-power microscope in combination with a stroboscopic light.

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[1] By M. S. Plesset, published in the September, 1949, issue of the Journal of Applied Mechanics, Trans. ASME, vol. 71, pp. 277–282.

[2] Assistant Professor of Hydraulics, St. Anthony Falls Hydraulic Laboratory, University of Minnesota, Minneapolis, Minn.,

[3] "Air Bubble Resorption," by Edward Silberman, Technical Paper No. 1, series B, St. Anthony Falls Hydraulic Laboratory, University of Minnesota, August, 1949.

The water in the cylinder was saturated with air at atmospheric pressure. Cavitation was produced in the rotating liquid by suddenly dropping the mean pressure only 1 or 2 psi. (A similar or much larger drop in pressure in static liquid produced no cavitation.) Immediately after the pressure drop very small bubbles began to form in the liquid. These were merely pin points of light of 0.001 in. diam or less. (The size estimate was obtained by stopping the rotor and watching a bubble in the microscope.) The bubbles continued to form as long as the low pressure was maintained. A further reduction in pressure (or an increase in the speed of rotation) increased the rate of formation of the small bubbles perceptibly, but did not increase their size to a measurable degree. A sudden increase in pressure did not reverse the process; a measurable time was required to make each bubble disappear. Of course the gas inside the bubbles was essentially air rather than water vapor, but the formation process was truly cavitation as it is usually defined. By reducing the saturation pressure of the liquid, small bubbles, consisting mainly of water vapor, could be produced just as well by the same process.

It might be well to mention at this point that the small bubbles were undoubtedly produced in the turbulent vortexes where the pressures were considerably smaller than 1 or 2 psi below atmospheric pressure. The bubbles were first visible at points about one quarter of the distance from the cylinder axis to its walls where the turbulence appeared to be greatest, and where the rotational and irrotational flow were joined. The small bubbles were probably expanded nuclei in accordance with the theory stated by the author; when once expanded, the nuclei maintained their size by virtue of their increased surface energy even though they moved into higher-pressure regions.

After initial formation of the small bubbles, the phenomenon observed by the writer was not cavitation as it is usually described by the engineer. As stated by the author and confirmed by many experimental investigations including those of the writer, the rate of diffusion of air into and out of the bubble is of a much smaller order of magnitude than the rate of evaporation or condensation of water vapor. Therefore as long as the existing pressure exceeds the vapor pressure, individual bubbles grow and shrink only in accordance with the gas laws as the pressure fluctuates rapidly. Incidentally, the principal method of growth of the writer's bubbles was by coalescence following collision, mainly along the cylinder axis.

It is believed that the presence of solid matter, the basis of the author's theory of bubble formation, was demonstrated in the writer's experiments. The distilled water, which was at first perfectly clear, came to have a dirty appearance after many repetitions of the cavitation process. The water was changed many times during the course of the experiments, but it always became dirty. This phenomenon was orginally attributed to wear on the seal where the rotor shaft entered the cylinder, but on dismantling the apparatus no signs of wear could be detected. It is now believed that the dirty appearance is attributable to the coalescence of submicroscopic solid nuclei from the coalescence of small bubbles over a period of time. In a few instances, solid particles were actually observed trapped in the surface of bubbles of about 0.02 in. diam. which had been formed by coalescence of many smaller bubbles.

Perhaps it should have been brought out in the paper that the occurrence of cavitation is not limited to the walls of a solid body, but that any discontinuity in the liquid is a potential cavitating surface by virtue of the vortexes generated thereat. The edges of wakes behind blunt bodies, the edge of the jet in an open-jet water tunnel, and the experiment just described by the writer are examples. The author's three regimes of flow apply to these examples as well as to the solid body.

The phenomenon of rebound, illustrated in Fig. 3 of the paper, requires much further study. Perhaps it may be explained in a homely way as the struggle of the liquid to release the surface energy of the bubble and to return its residual gas and vapor to the nuclear state of low surface energy.

Author's Closure

Mr. Silberman's remark that the occurrence of cavitation is not limited to the neighborhood of solid boundaries is correct, and it was not the author's intention to imply otherwise. It is well known that cavitation may occur in the reduced-pressure fields produced in vortex flow, but it is often difficult to get quantitative information from these experiments.

The author would like to comment on a suggestion of Mr. Silberman that a bubble when once expanded may maintain its increased size by virtue of its increased surface energy even though it moves into a region of higher pressure. An increase in surface energy, that is, the work done against surface tension, cannot in itself give a stabilizing effect on a gas bubble in a liquid; in any such configuration, surface tension always acts in the direction of contracting the bubble.

The author disagrees with Mr. Silberman's final remark that the rebound phenomenon is related in some way to the release of the surface energy of the bubble. The cavitation bubbles which we have analyzed collapse from a maximum radius of approximately 0.1 in. The ratio of the surface energy of such a bubble to the pressure potential energy is of the order of 10^{-3}. The surface energy therefore constitutes a trivial portion of the total energy. It is believed that the rebound phenomenon will be explained in terms of the compressibility effects which enter toward the end of the collapse.

An Energy Method for Determining the Dynamic Characteristics of Mechanisms[1]

W. M. Dudley.[2] This paper represents another worthwhile contribution by the author in the field of applying mathematics to the kinematic and dynamic analysis of engineering mechanisms. He shows that if a train of mechanism has rigid parts and only one degree of freedom, then in a given position the velocities of the parts bear fixed ratios to each other. These velocity ratios may be found throughout the motion cycle by the usual methods. At any position, each part contains a fixed percentage of the total kinetic energy. When these have been plotted, it becomes possible to solve two types of problems of practical importance: (1) If a curve of motion versus time is specified, one can find the driving-force function which is necessary to produce it; (2) if the driving force is specified, as by a spring or hydraulic cylinder, the resulting motion can be found.

The latter problem arises frequently in the engineering development of mechanisms. These are usually invented by toolmakers rather than by engineers. The toolmaker has a dual advantage in being skillful with his hands, and blissfully ignorant of high stresses, wear problems, the need for easy repair and adjustment, and other bugaboos which have a paralyzing effect on the thought processes of a trained engineer. When a machine has appeared which is workable on a laboratory basis, it is a very common experience that the inventor is unable to take the remaining steps necessary to make it practical for use without the continual presence of a skilled mechanic.

[1] By B. E. Quinn, published in the Journal of Applied Mechanics, September, 1949, Trans. ASME, vol. 71, pp. 283–288.

[2] Associate Director, Engineering Research Department, Standard Oil Company (Indiana), Chicago, Ill. Mem. ASME.