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SUBHARMONIC OSCILLATIONS IN SQUEEZE FILM DAMPED ROTOR BEARING SYSTEMS WITHOUT CENTRALIZING SPRINGS

Peter Y. P. Chen
School of Mechanical and Manufacturing Engineering
University of New South Wales
Sydney, NSW
Australia

E. J. Hahn
School of Mechanical and Manufacturing Engineering
University of New South Wales
Sydney, NSW
Australia

G. Y. Wang
School of Mechanical and Manufacturing Engineering
University of New South Wales
Sydney, NSW
Australia

ABSTRACT

This paper investigates the nonsynchronous motion of a rigid rotor in squeeze film damped systems without spring support subjected to both unbalance and unidirectional loading. Both harmonic balance and 4th order Runge–Kutta integration are used to obtain the unbalance response, the asymptotic stability of the harmonic balance equilibrium solutions being determined by Floquet theory. Depending on the system parameters stable subharmonic orbits with fundamental frequencies of half, one third and one quarter of excitation frequency were obtained. Also, bistable, tristable and quadristable solution possibilities were found. The effect of these subharmonic orbits on maximum transmissibilities is included.

NOTATION

$A_o^y, A_k^y, A_o^z, A_k^z$	Fourier coefficients
B_k^y, B_k^z	Fourier coefficients
C	radial clearance
$C_o^y, C_k^y, C_o^z, C_k^z$	Fourier coefficients
e, ε	journal eccentricity, $\varepsilon = e/C$
f_y, f_z	damper forces in y and z directions; $F_y = f_y/(mC\Omega^2)$ etc.
h	film thickness; $\bar{h} = h/C$
k	order of Fourier series component; $k = 1, \ldots n$
L	land width of the bearing
m	rotor mass per bearing station
n	highest order of truncated Fourier series
N	order of lowest significant subharmonic
p,q	$\bar{y}', \bar{z}'$
P	lubricant pressure
R	bearing radius
S_k^y, S_k^z	Fourier coefficients
t, τ	time; $\tau = \Omega t$
T	maximum transmissibility over period of equilibrium solution
U	unbalance parameter = ρ/C
W	unidirectional load parameter = $g/(C\Omega^2)$
Z	axial coordinate measured from bearing centre O_b in x direction as shown in Fig. 3; $\bar{Z} = Z/L$
x, y, z	coordinates located in centre plane of bearing at O_b as shown in Fig. 3; $\bar{y} = y/C, \bar{z} = z/C$
γ	a speed or frequency parameter; $\gamma = \omega/\Omega$
λ	$k\gamma/N$
μ	mean absolute viscosity of lubricant
ρ	mass eccentricity
ψ	angular location from $\hat{y}$ direction of A along bearing surface as shown in Fig. 3.
ψ^*	defined by eqn. (8)
Ω	angular velocity for non-dimensionalisation $= L^3\mu R/(mC^3)$
ω	angular velocity of rotor
$\dot{}, {}'$	denotes differentiation with respect to time t, τ

Subsubscript E denotes equilibrium value.

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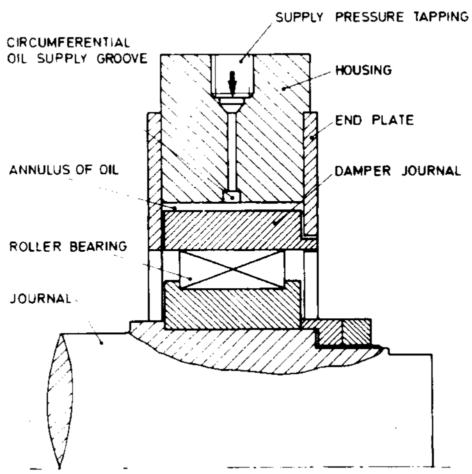

FIGURE 1. SCHEMATIC OF SQUEEZE FILM DAMPER

INTRODUCTION

Because of their constructional simplicity and wide range of damping capability, circular bore type squeeze film dampers have found frequent application in aeroengines, wherein the outer race of roller bearings forms the non-rotating journal surface of the damper. Depending on engine design, such dampers are generally either centrally preloaded (whereupon the gravitational load is balanced by retainer springs), or the centralizing springs are dispensed with altogether as in Fig. 1, relying on unbalance excitation for lift off. The former design is generally assumed to result in circular synchronous orbiting of the journal centre about the bearing centre and generalised techniques for determining all such equilibrium orbit solutions are available (Greenhill and Nelson, 1982; McLean and Hahn, 1983), which solutions need to be tested for asymptotic stability (McLean and Hahn, 1985). The unsupported damper design does not allow for such a simplifying assumption and has generally necessitated transient solutions (Mohan and Hahn, 1974) to obtain the steady state journal orbit, though recently, harmonic balance has been successfully applied to obtain stable equilibrium solutions for unsupported rigid rotors (Chen and Hahn, 1991; Wang and Hahn, 1991).

The possibility of stable nonsynchronous solutions for centrally preloaded dampers was conclusively demonstrated by Li and Taylor (1987) who found stable subsynchronous solution possibilities for rigid rotor systems with insufficient spring preload to centre the gravitational load. Such possibilities were confirmed by Zhao and McLean (1990) who used trigonometric collocation to evaluate the coefficients of the assumed periodic equilibrium solution. That such subharmonic solutions are possible is a consequence of the non-linearity of the system (Iwata and Kobori, 1981).

Though Li and Taylor (1987) and Zhao and McLean (1990) both assumed the presence of support springs, their results suggest that dampers without such springs could also experience subsynchronous solutions. Since harmonic balance enables stable equilibrium orbits for such unsupported dampers to be determined rapidly, regardless of the operating conditions, as distinct from the transient solution approach which is unsuited for such determinations under conditions of low damping, it is now opportune to investigate the operating conditions conducive to the existence of subharmonic solutions in unsupported dampers.

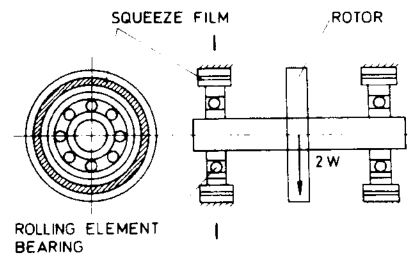

FIGURE 2. SYMMETRIC RIGID ROTOR WITH DAMPERS AT EACH END

THEORY

The relevant theory will be developed for a symmetric rigid rotor with an unsupported damper at either end as shown in Fig. 2. A section view through the centre of either dampers is given in Fig. 3.

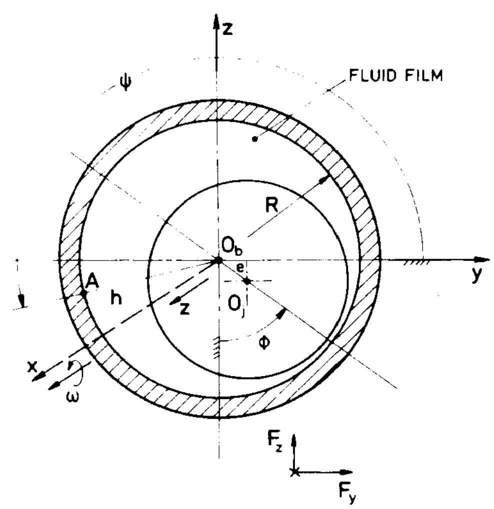

FIGURE 3. SECTION VIEW OF SQUEEZE FILM DAMPER SHOWING NOTATION

Assuming equivalent dampers at either end of the rotor and cylindrical rotor motions, the equations of motion at either damper station may be written as:

$$m\ddot{y} = m\rho\omega^2 \cos \omega t + f_y \qquad (1)$$

$$m\ddot{z} = m\rho\omega^2 \sin \omega t + f_z - mg \qquad (2)$$

or in non-dimensional form:

$$\bar{y}'' = U\gamma^2 \cos\gamma\tau + F_y \qquad (3)$$

$$\bar{z}'' = U\gamma^2 \sin\gamma\tau + F_z - W \; . \qquad (4)$$

The damper force components may be obtained by integrating the pressure distribution. Assuming that the Reynolds equation with constant fluid properties is applicable and that the short bearing approximation is valid, the pressure distribution in the damper is given by Chen and Hahn (1991):

$$P = \frac{6\mu L^2}{h^3}\left(\frac{1}{4} - \bar{Z}^2\right)\left(\dot{z}\sin\psi + \dot{y}\cos\psi\right) \qquad (5)$$

where $h = C - z\sin\psi - y\cos\psi$ (6)

and where the pressure at both ends of the bearing is assumed to be zero. Hence, one obtains:

$$\begin{Bmatrix} F_y \\ F_z \end{Bmatrix} = -\frac{RL}{mC\omega^2}\int_0^{2\pi}\left(\int_{-1/2}^{1/2} P\,d\bar{Z}\right)\begin{Bmatrix} \cos\psi \\ \sin\psi \end{Bmatrix} d\psi \qquad (7)$$

By setting all pressures below zero equal to zero, the integration of Eqn. (7) is considerably simplified, for the positive pressure region extends over an angular extent of exactly π. Thus, the ψ limits of integration in Eqn. (7) need only extend from ψ^* to $\psi^*+\pi$ where:

$$\tan\psi^* = -\dot{y}/\dot{z} \qquad (8)$$

with the value of ψ^* uniquely defined by the requirement that $\dot{y}\sin\psi + \dot{z}\cos\psi$ be positive for $\psi^* < \psi < \psi^* + \pi$. This cavitation condition was assumed in order to hasten computations. Insistence on zero pressure gradient normal to the cavitation boundary would considerably lengthen the computation with only a minor increase in accuracy and hence was not felt warranted. Eqn. (7) can then be integrated in the Z direction to give

$$\begin{Bmatrix} F_y \\ F_z \end{Bmatrix} = -\int_{\psi^*}^{\psi^*+\pi} \frac{\bar{z}'\sin\psi + \bar{y}'\cos\psi}{(1 - \bar{z}\sin\psi - \bar{y}\cos\psi)^3}\begin{Bmatrix} \cos\psi \\ \sin\psi \end{Bmatrix} d\psi \qquad (9)$$

Obviously, F_y and F_z are functions of the displacement and the velocity. Note that the solution of Eqns. (3) and (4) depend on the three non-dimensional parameters U, W and γ, so selected that only γ is speed dependent.

COMPUTATIONAL STRATEGY

Both transient and harmonic balance solution approaches were used to obtain the stable equilibrium solutions, the former serving mainly to seek out solution possibilities when no apparent stable equilibrium solutions could be located using harmonic balance. The harmonic balance approach, which is summarised in the appendix, assumes an equilibrium solution in the form of a Fourier series, so that the problem resolves into evaluating the unknown series coefficients from a set of as many nonlinear simultaneous equations as there are coefficients. The equilibrium solution so obtained then needs to be tested for asymptotic stability.

Both approaches have shortcomings. The transient solution approach, here 4th order Runge Kutta integration was used, becomes computationally too time consuming under conditions of low damping. The determination of whether a stable equilibrium solution has been reached, if there is one, can be rather tricky. Solutions can be initial value dependent, and there is no guarantee that all stable equilibrium solutions have been found.

The harmonic balance approach, on the other hand, is much faster, regardless of whether the system is lightly damped or not. However, it necessitates the simultaneous solution of a set of nonlinear equations, so that the solutions are both iterative technique and initial guess dependent. Again, one does not generally have the certainty that all possible equilibrium solutions have been found. A sufficient number of terms must be retained to ensure that the assumed Fourier series is convergent, and all relevant solution harmonics must be present in the initial guess. All equilibrium solutions needed to be tested for asymptotic stability.

By and large, the harmonic balance approach was used for the parameter range investigations, with the transient approach occasionally used to validate what appeared to be unexpected stable equilibrium solutions, as well as to seek out different stable equilibrium solution possibilities, and to study unstable equilibrium solution behaviour.

Once equilibrium orbits had been found, the maximum transmissibility during the fundamental period was calculated according to:

$$T = \text{Maximum value of } \left(F_y^2 + F_z^2\right)^{1/2} / \left(U\gamma^2\right). \qquad (10)$$

RESULTS

Equilibrium solutions were sought over the parameter range $0.1 < U < 0.5$, $0 < W < 1000$ and $\gamma > 0$. Only a few results are

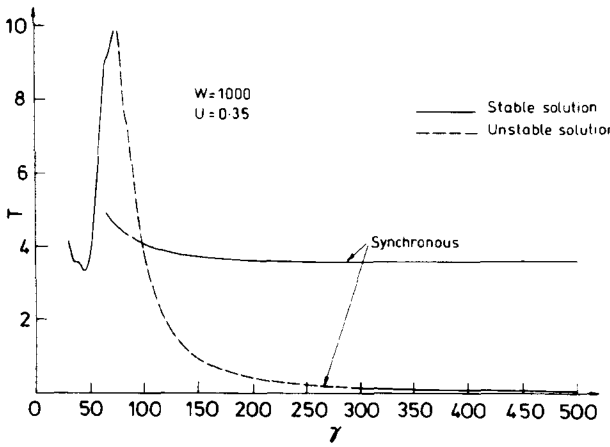

FIGURE 4. PLOT OF T VERSUS γ FOR U=0.35 AND W=1000

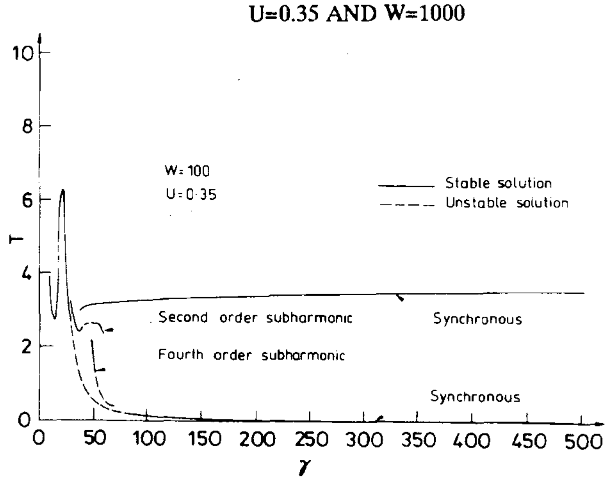

FIGURE 5. PLOT OF T VERSUS γ FOR U=0.35 AND W=100

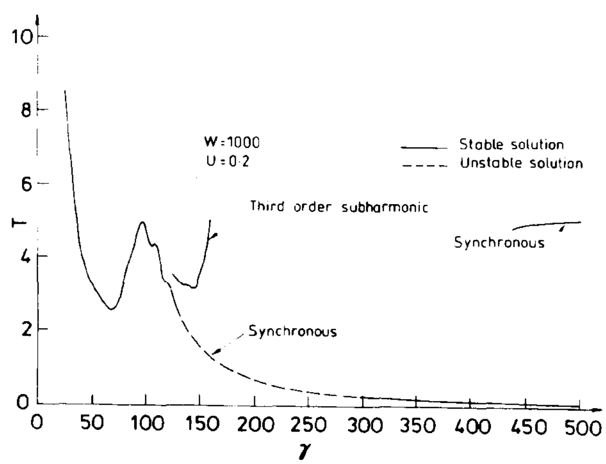

FIGURE 6. PLOT OF T VERSUS γ FOR U=0.2 AND W=1000

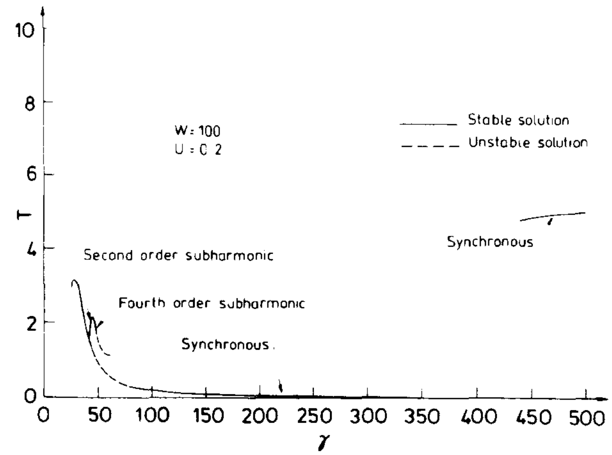

FIGURE 7. PLOT OF T VERSUS γ FOR U=0.2 AND W=100

presented here to illustrate the complexity and diversity of the solution possibilities. Figures 4 to 7 give results for T versus γ for U = 0.35 and U = 0.2 each for values of W = 1000 and W = 100. Full lines indicate stable equilibrium solutions.

Note that depending on the operating conditions, it was possible to find one or more stable equilibrium solutions, or only an unstable equilibrium solution. Thus, three stable equilibrium solutions are indicated for W = 100, U = 0.35 and $48 < \gamma < 50$. Indeed, for the parameter combination of W = 50, U = 0.375 and γ = 40, four stable equilibrium solutions were found and the steady state orbits for these four solutions are shown in Fig. 8. The occurrence of so many stable solutions was rare.

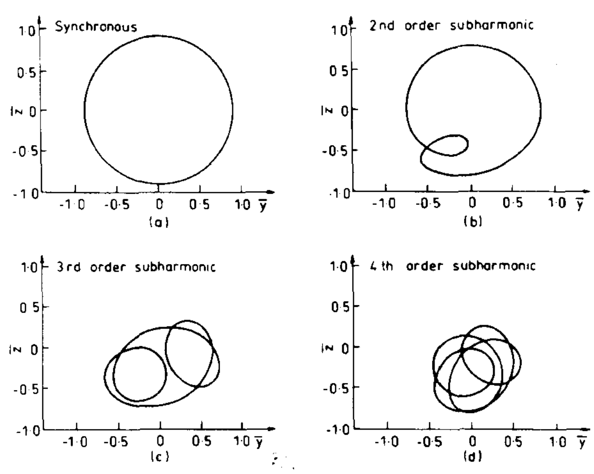

FIGURE 8. STABLE EQUILIBRIUM ORBIT FOR U=0.375, W=50 AND γ = 40

On the other hand, many parameter combinations were found for which no stable equilibrium solutions are indicated. Thus, in Fig. 6, for W = 1000 and U = 0.2, no stable solutions were found for $159 < \gamma < 300$. For $\gamma = 250$, Fig. 9 shows the last 10 cycles after 110 solution cycles. A plot of $\dot{z}$ versus z and $\dot{y}$ versus y, as suggested by Li and Taylor (1987) would indicate whether a high order subharmonic fundamental exists. Even after 50 cycles (after an initial 200 cycles) no subharmonic fundamental could be located. Generally, harmonic balance investigations did not seek equilibrium solution possibilities below fourth order subharmonics, owing to the rapid increase in the number of Fourier coefficients required for accuracy as the order of the subharmonic increases.

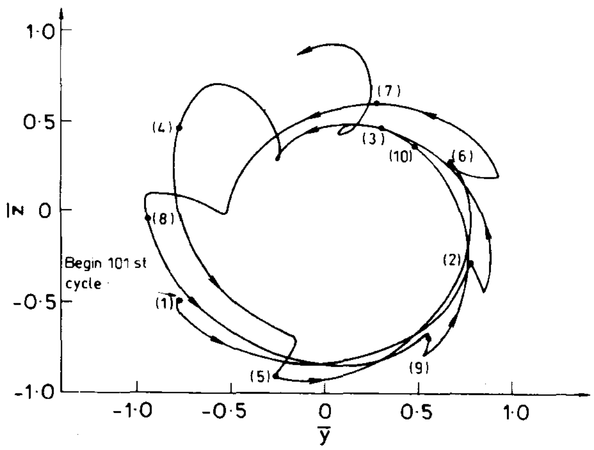

FIGURE 9. UNSTABLE EQUILIBRIUM ORBIT FOR U=0.2, W=1000 AND γ = 250

Some trends are apparent even from the limited data presented. Thus, as the unbalance increases, the possibility of undesirable multistable operation is increased. Referring to Fig. 7, for W = 100, and U = 0.2, the damper operates with low T over the speed range $75 < \gamma < 435$. Once γ exceeds 435, a huge increase in T is possible. Subsynchronous vibration, or unstable behaviour occurs only for γ less than 75, at which relatively low speeds, high transmissibilities are not all that relevant as the dynamic forces themselves are low. As the unbalance increases, this desirable operation regime is progressively reduced and as seen in Fig. 5, is non-existent when U = 0.35, by which time, high T is possible over the whole operating speed range. As may be seen from Figs. 4 and 6, similar predictions pertain to W = 1000, except in this case, the speed range over which no stable solutions were found has increased considerably. Such unstable regions are more likely at lower unbalance values, for as unbalance increases, the high T stable synchronous solution occurs at lower γ values and stabilises the situation, so to speak. Thus, for U = 0.2, the range for unstable only solutions has shrunk from $159 < \gamma < 300$ to $48 < \gamma < 75$ as W decreases from 1000 to 100.

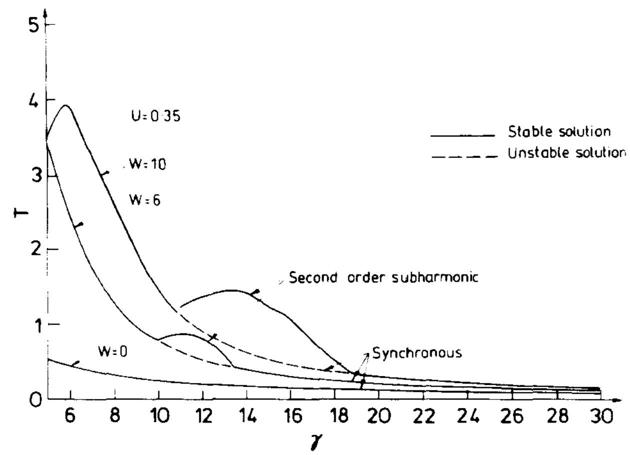

FIGURE 10. PLOT OF T VERSUS γ FOR U=0.35 AND 0<W<10

Subharmonic solutions tend to be restricted to relatively low γ values, depending on W and U. Though no subharmonic solutions are indicated for W = 1000 and U = 0.35 (Fig. 4) some have been found for W = 1000 and U = 0.2 (Fig. 5) and an upper bound to W at which no subharmonic solutions exist regardless of U has not yet been found. However, as W decreases, for some given U, the speed range for subharmonic possibilities tends to decrease and, as may be seen in Fig. 10, disappears at W = 0. Nor were subharmonic solutions found at W = 0 for other U values either. This agrees with the observations of Li and Taylor (1987), for spring supported dampers. In such cases, all orbits were circular. Hence, it may be concluded that a resultant unidirectional loads is necessary to produce subharmonic orbits.

SUMMARY OF CONCLUSIONS

1. Neither the transient nor the harmonic balance approach can guarantee that all possible stable equilibrium solutions have been found. Both approaches were occasionally needed to seek out stable equilibrium solutions.
2. A resultant unidirectional load appears necessary for subharmonic equilibrium solutions. With such a load, depending on operating conditions, significant second, third or fourth order subharmonic components were found.
3. Depending on operating conditions, no stable equilibrium solution or as many as four stable equilibrium solutions could be found.

4. The possibililty of undesirable multistable operation, with high maximum transmissibility increases with increased unbalance parameter.
5. Subsynchronous vibrations tend to occur at relatively low values of the speed parameter, and even though they involve high maximum transmissibilities, the dynamic forces are expected to be low.
6. Unstable regions, ie regions for which no stable equilibrium solutions have been found, tend to be favoured by high unidirectional loading and low unbalance loading.

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APPENDIX

The Harmonic Balance Approach

Equilibrium Solutions. If steady state conditions have been reached, one can assume solutions to eqns. (3) and (4) of the form:

$$\bar{y}_E = A_o^y + \sum_{k=1}^{n}\left(A_k^y \cos\lambda_k\tau + B_k^y \sin\lambda_k\tau\right) \tag{A1}$$

$$\bar{z}_E = A_o^z + \sum_{k=1}^{n}\left(A_k^z \cos\lambda_k\tau + B_k^z \sin\lambda_k\tau\right) \tag{A2}$$

where $$\lambda_k = k\gamma / N \tag{A3}$$

and where it is assumed that there are n harmonics of the fundamental frequency ω/N; i.e. an integer value of N allows for the possibility of an N'th order subharmonic of the excitation frequency ω. Eqns. (3) and (4) then become:

$$\bar{y}_E'' = U\gamma^2 \cos\gamma\tau + F_{y_E} \tag{A4}$$

$$\bar{z}_E'' = U\gamma^2 \sin\gamma\tau + F_{z_E} - W \tag{A5}$$

Since F_{y_E} and F_{z_E} are functions of $\bar{y}_E, \bar{y}_E', \bar{z}_E$ and $\bar{z}_E'$, they themselves must be periodic with the same frequency components as the $\bar{y}_E$ and $\bar{z}_E$, i.e.:

$$F_{y_E} = C_o^y + \sum_{k=1}^{n}\left(C_k^y \cos\lambda_k\tau + S_k^y \sin\lambda_k\tau\right) \tag{A6}$$

and $$F_{z_E} = C_o^z + \sum_{k=1}^{n}\left(C_k^z \cos\lambda_k\tau + S_k^z \sin\lambda_k\tau\right) \tag{A7}$$

where by definition,

$$C_k^y = \frac{1}{\pi}\int_0^{2\pi} F_{y_E} \cos(\lambda_k\tau)\, d\tau = C_k^y\left(\bar{y}_E, \bar{y}_E', \bar{z}_E, \bar{z}_E'\right) \text{etc.} \tag{A8}$$

Satisfaction of eqns. (A4) and (A5) at all equilibrium solution times requires that for all k:

$$0 = C_o^y \tag{A9}$$

$$-\lambda_k^2 A_k^y = C_k^y + \beta U \gamma^2 \tag{A10}$$

$$-\lambda_k^2 B_k^y = S_k^y \tag{A11}$$

$$0 = C_o^z - W \tag{A12}$$

$$-\lambda_k^2 A_k^z = C_k^z \tag{A13}$$

$$-\lambda_k^2 B_k^z = S_k^z + \beta U \gamma^2 \tag{A14}$$

where $\beta = 1$ when $k = N$, and $\beta = 0$ otherwise.

Thus, the equation set (A9) to (A14) constitutes a set of (4n + 2) non-linear simultaneous equations in the (4n + 2) unknowns $A_o^y, \ldots. A_n^y, B_1^y, \ldots. B_n^y, A_o^z, \ldots. A_n^z, B_1^z, \ldots. B_n^z$.

Stability of Equilibrium Solutions:

Consider small motion perturbations to the equilibrium solutions $\bar{y}_E$ and $\bar{z}_E$, so that eqns. (3) and (4) are relevant. Subtracting eqns. (A4) and (A5) respectively yields:

$$\bar{y}'' - \bar{y}_E'' = \Delta\bar{y}'' = F_y - F_{y_E}$$

$$= \frac{\partial F_y}{\partial \bar{y}}\Delta\bar{y} + \frac{\partial F_y}{\partial \bar{z}}\Delta\bar{z} + \frac{\partial F_y}{\partial \bar{y}'}\Delta\bar{y}' + \frac{\partial F_y}{\partial \bar{z}'}\Delta\bar{z}' \tag{A15}$$

$$\bar{z}'' - \bar{z}_E'' = \Delta\bar{z}'' = F_z - F_{z_E}$$

$$= \frac{\partial F_z}{\partial \bar{y}}\Delta\bar{y} + \frac{\partial F_z}{\partial \bar{z}}\Delta\bar{z} + \frac{\partial F_z}{\partial \bar{y}'}\Delta\bar{y}' + \frac{\partial F_z}{\partial \bar{z}'}\Delta\bar{z}' \tag{A16}$$

For asymptotic stability, $\Delta\bar{y}$ and $\Delta\bar{z}$ must approach zero with time. Since eqns. (A15) and (A16) constitute a set of linear differential equations with periodic coefficients of period $2\pi N/\gamma$, Floquet theory (Coddington and Levinson, 1955) may be conveniently used to test for stability. Thus, letting $p = \bar{y}'$ and $q = \bar{z}'$, eqns. (A15) and (A16) may be written as:

$$\begin{bmatrix} \Delta\bar{y}' \\ \Delta\bar{z}' \\ \Delta p' \\ \Delta q' \end{bmatrix} - \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\partial F_y}{\partial \bar{y}} & \frac{\partial F_y}{\partial \bar{z}} & \frac{\partial F_y}{\partial p} & \frac{\partial F_y}{\partial q} \\ \frac{\partial F_z}{\partial \bar{y}} & \frac{\partial F_z}{\partial \bar{z}} & \frac{\partial F_z}{\partial p} & \frac{\partial F_z}{\partial q} \end{bmatrix} \begin{bmatrix} \Delta\bar{y} \\ \Delta\bar{z} \\ \Delta p \\ \Delta q \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{A17}$$

Let G be the 4 x 4 matrix whose columns contain the four solutions at time $\tau = 2\pi N/\gamma$ of the above equations, having as initial conditions the corresponding column of the 4 x 4 identity matrix. Then the system is stable if all the eigenvalues of G have magnitudes less than unity. These four solutions may be obtained by any technique whatsoever, eg 4th order Runge Kutta with variable step size was used in this work.