THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS
345 E. 47 St., New York, N.Y. 10017

87-GT-231

Design Criteria and Efficiency Prediction for Radial Inflow Turbines

ANTONIO PERDICHIZZI
Associate Professor
Energetics Department
Politecnico di Milano
Milan, Italy

GIOVANNI LOZZA
Research Engineer
Hydraulic Eng. Department
University of Pavia
Pavia, Italy

ABSTRACT

A theoretical investigation was performed to predict the maximum achievable efficiency of radial inflow turbines for different design conditions. The analytical tool used in the investigation is a computer code able to perform the contemporary optimization of the main design variables, in order to obtain maximum efficiency. Since the results are strictly dependent on the loss correlations, reliability of the efficiency predictions was tested at first by comparison with several test-cases available in literature: good agreement with experimental data was found, pointing to the validity of the results presented here.

A large number of cases were analyzed: the efficiency and the main design parameters, obtained after the optimization process, are presented for optimum specific speed. Turbine efficiency was found to be dependent both on compressibility effects, related to the volume expansion ratio, and on actual turbine size, accounting for geometric non-similarity effects. Influence of non-optimum specific speed is also discussed.

By means of similarity rules, the results enable turbine design to be performed in a simple way, for a variety of working fluids and design conditions.

NOMENCLATURE

c : blade chord, m
D : turbine diameter, m
h : enthalpy, J/kg
H : blade height, m
k : blade surface roughness, m
K_{is} : head coefficient, $2\Delta h_{is}/u_1{'}^2$
L_{cl} : energy loss, caused by the tip clearance, J/kg
L_{df} : energy loss, caused by the disk friction, J/kg
M : Mach number
MM : fluid molecular mass
N_s : specific speed, $(RPM/60)\sqrt{V_{2is}}/\Delta h_{is}^{3/4}$
o : blade throat opening, m
p : pressure, Pa
r^* : isentropic degree of reaction
R : turbine radius, m
Re : Reynolds number
RPM : speed of revolution, rpm
t : blade trailing-edge thickness, m
T : temperature, K
u : peripheral speed, m/s
v : absolute velocity, m/s
V : volumetric flow rate, m^3/s
VH : size parameter, $\sqrt{V_{2is}}/\Delta h_{is}^{1/4}$, m
VR : volume expansion ratio, V_{2is}/V_0
w : relative velocity, m/s
Z : number of blades
α : absolute flow angle, relative to the tangential direction, deg
β : relative flow angle, relative to the tangential direction, deg
γ : fluid specific heat ratio
δ_r : tip clearance, m
Δ : variation
Π : expansion ratio, p_{T0}/p_{2m}
μ : fluid viscosity, Pas
Φ_E : kinetic energy recovery coefficient
ζ : loss coefficient
η : turbine efficiency
ω : speed of revolution, rad/s
κ : exponent used in the radial equilibrium law

Subscripts

h : hub of the blade
is : isentropic process
m : mean radius at rotor exit
R : relative to the rotor
s : shroud of the blade
S : relative to the stator
scr : relative to the scroll

Presented at the Gas Turbine Conference and Exhibition, Anaheim, California — May 31-June 4, 1987

t : tangential component
T : total condition
TS : total-to-static
0 : stator inlet
1 : stator outlet
1' : rotor inlet
2 : rotor outlet

1. INTRODUCTION

In designing thermodynamic energy conversion systems, the choice of turbomachinery configuration is a very important step. Radial inflow turbines (RIT) can be employed, instead of axial-flow ones, in a wide range of cases, especially for low power range and when a large number of stages is not required. Typical applications are small gas turbines, steam turbines for cogeneration and Organic Rankine Cycle expanders.

For a preliminary evaluation of the potential of radial inflow turbines, a simple method to predict the achievable efficiency, without carrying out a detailed aerodynamic design, can be useful. Such a method, for different expansion ratios and actual machine size, is presented in this paper.

The aerodynamic design procedure of a turbomachine can be schematized in the following steps: (i) selecting the velocity triangles, (ii) selecting the speed of revolution, (iii) selecting the blade geometry, (iv) loss evaluation. If the above steps are worked out separately, it is practically impossible to achieve an actual optimization of the machine, as the mutual interaction among the involved parameters is not accounted for. So, a computer code, capable of performing a multi-variable contemporary optimization procedure, was developed. Such a code performs iterative turbine calculations, varying all the independent variables; the search of the maximum efficiency is based on the partial derivative analysis of the goal function, with respect to the independent design variables. The same approach was previously used for axial-flow turbines by Macchi and Perdichizzi (1981).

The fluid dynamic analysis is one-dimensional in the stator and at the rotor inlet while the simple radial equilibrium equation is solved at the rotor exit in order to avoid over-simplyfied calculations. This phase is the first step in the fluid dynamic design procedure, but it is of the utmost importance, since geometry and fluid velocities are then completely defined; the more sophisticated computations carried out in the subsequent design phases will provide a better understanding of the flow conditions and the optimum profile shapes, but not a substantial gain in efficiency. As far as the loss correlations are concerned, an effort has been made to adopt the most comprehensive ones, and to test their reliability by comparing them with experimental data: in fact, the uncertainty in losses evaluation is the most questionable item in the whole approach used in the paper. Yet, the satisfying agreement found in each test case is confirmation of the validity of the presented results.

2. CALCULATION METHOD

2.1 Flow analysis

The main fluid dynamic quantities involved in the turbine design and in the losses evaluations, are calculated in the following sections:

- Section 0, stator inlet. The scroll is supposed to be designed to mantain circumferentially constant conditions at the stator inlet. The radial component of the velocity is calculated by the continuity equation; the tangential one is imposed by momentum conservation between scroll inlet and stator inlet. Small scroll sections with small stator blade deflections, but large friction losses, are the consequence of adopting high tangential velocities.
- Section 1, stator exit. The stator profile is straight from the throat to the trailing edge and circular from the leading edge to the throat; zero incidence is assumed at the inlet section. The flow deviation is evaluated by correlations currently used in axial machines (Ainley and Mathieson, (1951) for subsonic flow, or Vavra (1969) for supersonic flow); the blade shape is convergent-divergent in the case of Mach number higher than 1.4.
- Section 1', rotor inlet. The flow is still supposed one-dimensional and momentum conservation is imposed. Incidence losses are computed by the dissipation of the relative tangential kinetic energy at rotor inlet, as suggested by Balje'(1952) and Benson (1970). Blade inlet angle can be arranged in accordance with optimal incidence criteria (i.e. slip factor, Wiesner,1967).
- Section 2, rotor exit. In this section radial inflow turbines have a very high blade height with respect to the mean diameter, i.e. low hub/tip ratios. With such geometries a one-dimensional approach is over-simplified, and does not give any information on the radial development of the blade exit angle. So the simple radial equilibrium equation (SREE) is solved in the form: $(dp/dR)= \rho(v_t^2/R)$, accounting for the entropy variations, but not for streamline curvature. The following iterative scheme is used:
  - a) for given β_{2m} and R_{2m}, a first tentative blade height H_2 is imposed;
  - b) for each radius, total relative pressure and temperatures are calculated;
  - c) the velocity triangles at mean-line are calculated by imposing ζ_R;
  - d) computation of ζ_R by the loss correlation;
  - e) if ζ_R is different from the imposed one, repeat from c);
  - f) a tentative distribution of density at each radius is imposed;
  - g) the static pressure at each radius is calculated by numerical integration of the SREE, after the evaluation of v_t;
  - h) velocities and thermodynamic quantities are calculated at each radius;
  - i) if the calculated densities differ from the imposed ones, repeat from g);
  - l) the mass flow is calculated by integrating the continuity equation;

m) if the calculated mass flow differs from the real one, repeat the procedure from b), assuming a new H_2.

It has to be pointed out that step (g) requires knowing v_t at each radius, for a known v_{tm}. Usually a design criterion is arbitrarily imposed, such as free vortex (Rv_t=const.) or constant blade angle. In the present case it is difficult to find a general rule: for example, unacceptable geometries can be found for very high specific speeds, using a free vortex blading design. In other cases, an exducer with constant blade angle can cause an excessive amount of leaving energy losses, especially when D_{2s}/D_{2h} is high. Therefore:

$$R^{\kappa} v_t = \text{const.}$$

was imposed, where the exponent κ is a variable to be optimized in order to get a satisfactory blade angle distribution for each case. Rotor exit flow deviation was estimated by means of the sine rule.

2.2 Loss evaluation

Since for radial inflow turbines a comprehensive method for loss evaluation does not exist in the open literature, correlations from different authors were used. This procedure is somewhat arbitrary; therefore the reliability of the loss prediction method was extensively tested, and the results are presented in the next paragraph. Let us now discuss the various sources of loss:

- Scroll losses: the friction coefficient, as a function of the Reynolds number and of the surface roughness, is computed at various hydraulic diameters.
- Stator losses: the correlation by Craig and Cox (1971) was used on the equivalent axial cascade obtained, as suggested by Vavra (1968), by a conformal transformation of the actual radial cascade, thus ensuring equivalent geometry and velocity distribution. This procedure implies a certain degree of uncertainty, especially as far as the secondary losses are concerned, but a complete correlation was preferred to oversimplified ones, often used for RIT.
- Vaneless losses: the correlation by Khalil-Tabakoff (1976) correlation was used, which accounts also for the compressibility effects.
- Rotor losses: estimating the rotor loss coefficient is the most difficult problem for RIT efficiency prediction. Preliminary calculations were performed using the correlations proposed by Glassman (1976), by Bridle and Boulter (1967), by Balje' (1952) and by Futral and Wasserbauer (1965) (the last three ones as suggested by Benson, 1970). Only the first one (Glassman, 1976) gave reliable results, compared with available data. This correlation is based on a simple evaluation of the boundary layer thickness development, corrected by the influence of finite trailing edge thickness; secondary flows effects are accounted for by calculating the ratio between 3D and 2D wetted surfaces; blade load is not considered, and the number of blades is calculated by a correlation proposed by the same author. Therefore, the Glassman correlation has been adopted for the following reasons:
  - it was formulated on the basis of the experimental activity carried out by NASA for radial turbines development (Kofskey and Holeski, 1965, Wasserbauer and Kofskey, 1966, Kofskey and Wasserbauer, 1966,1968 and 1969) which nowadays provides the major source of experimental data about RIT;
  - it is the most complete correlation among the ones available in literature, since it takes into account all the parameters involved in the loss generation;
  - it provided the best results in the comparison with the experimental data analyzed in the next paragraph.

  It has to be pointed out that Glassman uses a "reference loss coefficient", corrected by C_R for calibrating the losses with available results. The value of this coefficient was assumed equal to unity, as proposed by the author: this assumption is believed "to be representative of the state of the art performance for carefully designed turbines" (Glassman, 1976). In the optimization procedure, this value has not been varied: in fact, although the efficiency of tentative turbines might be slighty overestimated, it does not affect the final result, since the optimized turbine is always supposed to be carefully designed.

  Moreover, it is impossible to predict in a simple way a correct value of C_R for unconventional geometries.
- Leakage losses and disk friction losses: the method of Glassman (1976) has been used.
- Leaving kinetic energy losses: they are evaluated by integrating the radial distribution of the absolute velocity. Eventually the presence of a diffuser is accounted for by introducing the kinetic energy recovery coefficient Φ_E.

2.3 Optimization procedure and constraints

The optimization process requires defining the following items:

1) function to be optimized:
it consists of the turbine efficiency, defined as:

$$\eta = \frac{u_1 v_{1t} - \tilde{u}_2 \tilde{v}_{2t} - L_{df} - L_{cl}}{\Delta h_{is,TS} - \Phi_E(\tilde{v}_2^{\,2}/2)}$$

where the superscript means that the quantities are mass-averaged at the rotor exit, using as mass the stream tube mass flow.

2) variables to be optimized:
they are listed in Tab. 1A. The numerical optimization routine provides the combination of the independent variables, which correspond to the maximum efficiency; this is obtained by an iterative process based on incremental analysis. Usually, 2000 ∿ 3000 iterations are required, corresponding to 200 CPU seconds on Gould 67/32 computer (10 seconds on Sperry 1100/90 mainframe).

3) constraints:
several constraints are imposed, as shown in Tab. 1D, because: (i) it is necessary to avoid unrealistic values for the variables, (ii) the solution has to remain within the limits of validity of the correlations, (iii) the practical feasibility and the structural integrity must be ensured with

A. OPTIMIZING VARIABLES

K_{is}, r^*, R_{scr}/R_0, α_1, Z_S, R_1/R_1', R_{2m}/R_1', β_{2m}, ω, κ

B. FIXED INPUT VARIABLES (*)

$k = 5 \cdot 10^{-6}$ m $\qquad \delta_R = \max(0.3$ mm, $H_R/50)$

$t_S = \max(0.3$ mm, $o_S/20)$ $\qquad t_R = \max(0.5$ mm, $H_R/50)$

C. DESIGN INPUT DATA

p_{T0}, T_{T0}, p_{2m}, $\dot{m}$, MM, γ, μ

D. CONSTRAINTS (*)

independent variables					dependent variables				
1.3	<	K_{is}	<	5	.0015	<	o_S	<	0.05
0.3	<	r^*	<	0.8	0	<	u_1'	<	u_{max}
0.15	<	R_{scr}/R_0	<	0.8	20°	<	β_{2h}	<	70°
13°	<	α_1	<	60°	20°	<	β_{2s}	<	77°
6	<	Z_S	<	30	0.4	<	R_{2s}/R_1'	<	0.9
1.02	<	R_1/R_1'	<	1.1	0.3	<	R_{2h}/R_{2s}	<	0.8
0.25	<	R_{2m}/R_1'	<	0.75	0	<	M_{w1}	<	0.8
20°	<	β_{2m}	<	70°	13°	<	α_0	<	90°
0	<	κ	<	1.3	0	<	w_1	<	w_2

Tab.1: Optimizing variables, fixed input variables, design input data and constraints used in the optimization procedure.

(*) actual values have been used in the comparison with test cases.

usual manifacture technologies and materials.

3. Comparison with experimental results

In the above paragraphs, it was pointed out that reliability of the loss correlations is somewhat questionable. Since the results of the optimization are dependent on loss evaluation, a comparison with experimental and theoretical investigations was considered necessary to support the validity of the results.

A number of test cases, within a broad range of operating conditions, was selected. All the considered test cases are derived from experimental investigations, except for the work of Civinkas and Povinelli (1984) which is a theoretical work: it was considered since it provides a detailed internal loss repartion, useful for comparison with present predictions.

Computed and experimental efficiencies are shown in Tab.2, together with the main features of the analyzed turbines. Agreement is reasonably good,with calculated values generally lower than the experimental ones by 1-2 efficiency points. The original design of the test-cases was then optimized under the same operating conditions; while marginal improvements were obtained for the turbines D,E and F (Tab.2), in all other cases a gain of 4-7 efficiency points was obtained, chiefly due to a reduction of the degree of reaction.

A more detailed analysis was performed for cases C and D, for which a loss assessment was available. As shown in Fig.1, also the single sources of loss seem to be well predicted. The optimization leads to the

Case	Authors	η_{TS} exp.	η_{TS} calc.	Π_{TS}	N_s	K_{is}	r^*	v_{2t}/u_2	R_{2s}/R_1'	R_{2h}/R_{2s}	H_s/D_1'
A	Wasserbauer,	.810	.827	1.54	.111	2.06	.580	-.031	.711	.350	.118
	Kofskey, 1963		.880	"	"	2.27	.448	.124	.897	.307	.142
B	Kofskey,	.830	.828	1.54	.111	2.06	.580	-.020	.711	.350	.118
	Holeski, 1965		.886	"	"	2.14	.450	.177	.900	.368	.134
C	McLallin,	.788	.769	3.25	.071	2.69	.580	.025	.627	.484	.054
	Haas, 1980		.855	"	"	2.64	.430	.313	.737	.414	.057
D	Civinkas, (*)	.914	.879	3.49	.073	2.18	.450	.212	.650	.515	.086
	Povinelli,1984		.891	"	"	2.14	.468	.267	.698	.315	.041
E	Sasaky et al.,	.890	.867	2.23	.103	2.09	.460	.300	.654	.143	.079
	1977		.898	"	"	2.21	.460	.280	.896	.290	.100
F	Northern	.889	.871	2.06	.097	1.93	.670	.290	.681	.393	.083
	Research, 1983		.882	"	"	1.84	.670	.360	.690	.297	.088
G	Northern	.854	.832	3.50	.073	2.45	.530	-.060	.526	.423	.056
	Research, 1983		.880	"	"	2.37	.426	.335	.613	.315	.047

Tab.2: Efficiencies and main turbine features of the considered test cases. For each case, the first line values are estimated by the present method from the original geometry; the second line values refer to the optimized turbine.

(*) This is a theoretical work; η and Π are not total-to-static, but total-to-total, in agreement with the values quoted by the authors.

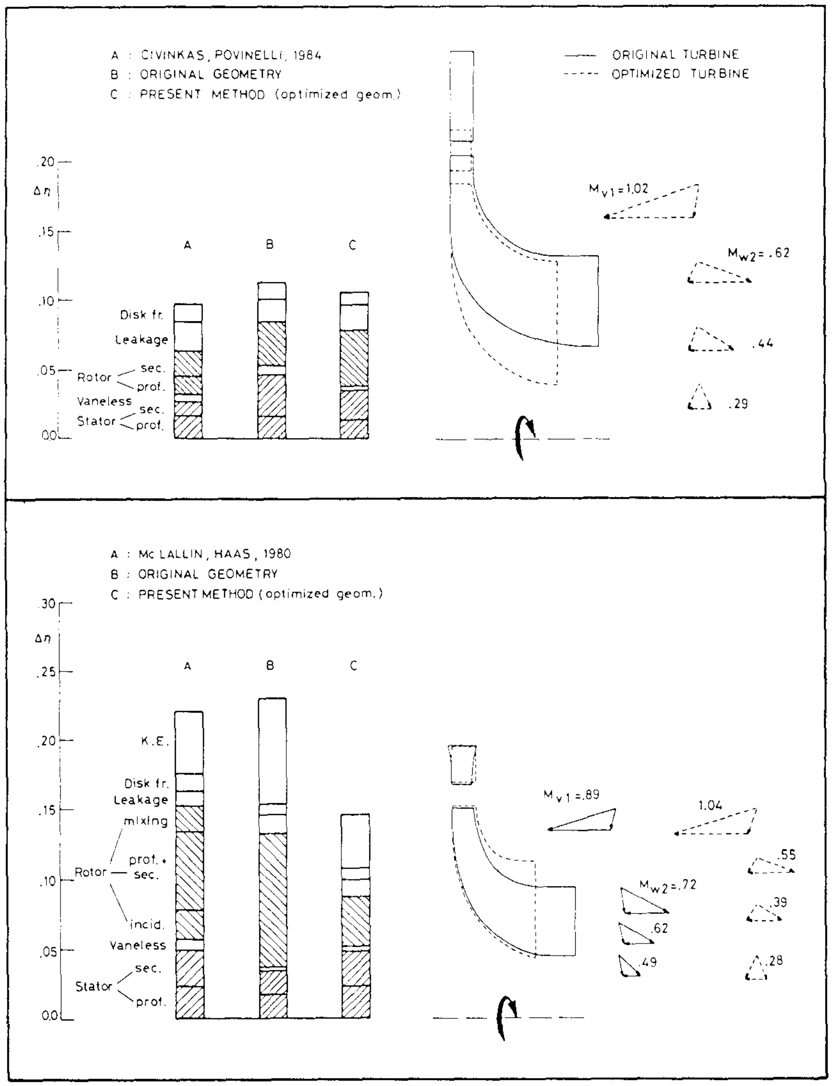


Fig.1: Efficiency penalties, merdional geometries and velocity triangles for the test-cases of Civinkas and Povinelli, 1984, and McLallin and Haas, 1980.

following results: (i) for case D both the geometry and the loss assessment do not present sensible variations, (ii) for case C an important reduction of the rotor losses takes place, caused by a larger β_2 and an higher swirl coefficient; also the leaving losses are reduced, by an increase of the exit area.

As a conclusion it can be said that (i) losses are well predicted, within the accuracy that can be expected when using generalized loss correlations, and (ii) the optimization procedure seems to lead to reasonable solutions with better efficiencies.

4. THE USE OF SIMILARITY PARAMETERS

According to similarity rules, the results obtained for a particular turbine can be extended to other turbines operating at the same specific speed, provided that:

1) the turbines are rigorously similar, including surface roughness, clearance and trailing edge thickness. Normally, this condition is not satisfied when large size variations are considered: for this reason, small turbines suffer significant efficiency penalties. Therefore, the dimensional parameter VH was introduced in order to identify the turbine "size"; it means that turbines with the same VH, have the same actual dimensions.
2) the working fluid has the same expansion ratio and the same heat capacity ratio, yielding the same specific volume variation and Mach number. In order to extend the validity of the results to working fluids with different thermodynamic behavior, the volume expansion ratio VR was used instead of the pressure ratio; in fact (Macchi and Perdichizzi, 1981) for a constant VR, fluids with very different heat capacity ratio yield almost similar Mach number and equal area variations through the turbine.
3) Reynolds number effects are neglected. In this analysis, Re is assumed constant ($5 \cdot 10^5$), so that the losses depend only upon the relative surface roughness. For lower Re, the efficiency predicted by this method should be corrected by means of published correlations.

With these assumptions, the most commonly used parameters for describing the turbine characteristics are the specific speed and the specific diameter. The specific diameter is not taken as an independent variable, but as an optimization result: that is, all the results are valid at optimum specific diameter. This procedure is correct when small enthalpy drops are considered (low expansion ratios and/or working fluids having high molecular weight); otherwise, mechanical stress limitations can impose diameters lower than optimum: in such conditions severe efficiency penalties result. The problem is not discussed here; some indications can be found in Lozza et al. (1986).

5. RESULTS AT OPTIMUM SPECIFIC SPEED

In turbine design, specific speed plays a fundamental role in the achievement of maximum efficiency. An appropriate selection of the optimum specific speed can be made using Fig.2: it is shown that it depends upon both compressibility and machine size. For large VR, the optimum N_s decreases, in order to provide larger diameters and exit areas and to limit the exit kinetic energy. Low N_s are required also for small turbines (VH<.02), to reduce the enhanced importance of the leakage losses.

The maximum achievable total-to-static efficiency is presented in Fig.3. Significant efficiency penalties occur at large VR: this is mainly due to the very small

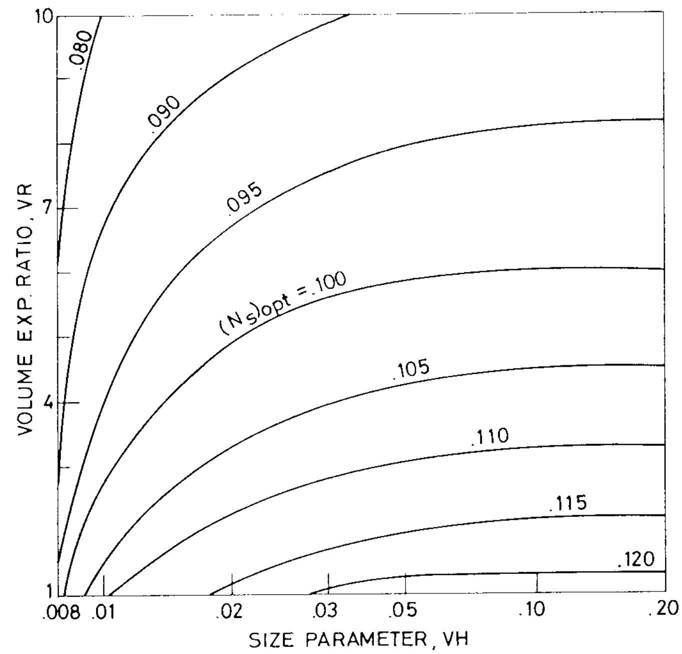


Fig.2: Optimum specific speed, as a function of VH and VR.

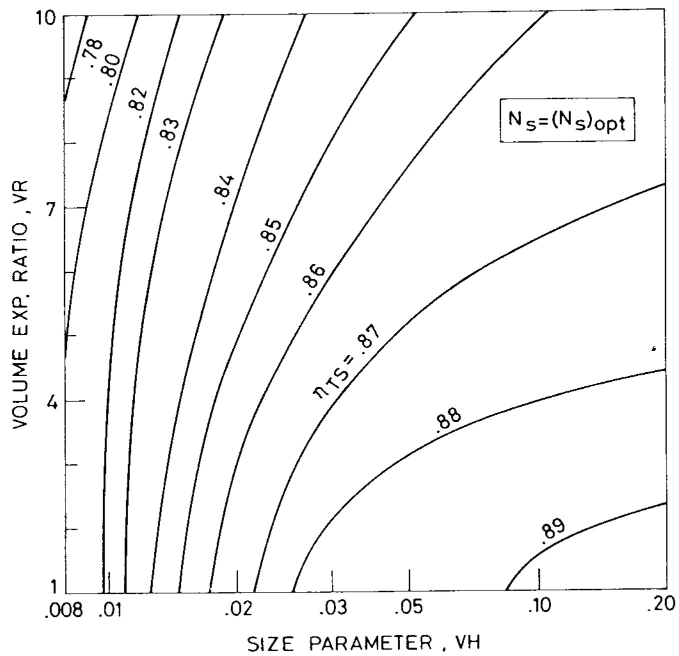
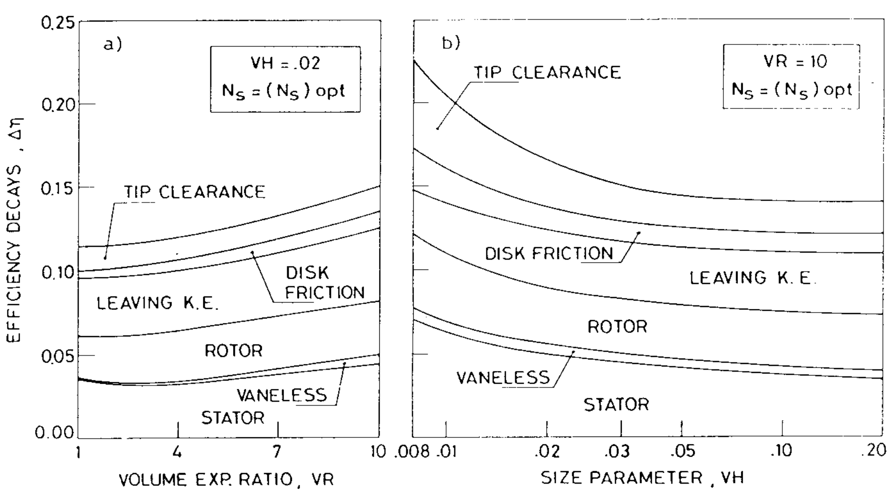


Fig.3: Efficiency prediction for optimum specific speed.

blade height of the stator and at the rotor inlet, because of the very low volume flow rate, in respect to the one at the rotor exit. Nevertheless, compressibility does not produce dramatic effects, as in the case of axial-flow turbines (Macchi and Perdichizzi, 1981), for the following reasons: (i) it is much easier to dimension rotor exit areas large enough to limit the kinetic energy losses, (ii) lower Mach numbers are generally common place, both in the stator and in the rotor, since a substantial portion of the enthalpy drop ends as centripetal work ($\Delta u^2/2$); in this way no additional shock losses are produced. This is confirmed by the loss distribution vs. VR (Fig.4a): the stator and vaneless losses increase because of the smaller blade heights, which are also responsible for the higher clearance and disk friction losses.

Larger efficiency penalties (up to 8 points) are predicted for very small turbines: in Fig.4b, it can be seen that the leakage losses dramatically increase by scaling down the turbine, as well as the stator loss, because of trailing edge and surface roughness effects: rotor losses do not vary significantly, because they



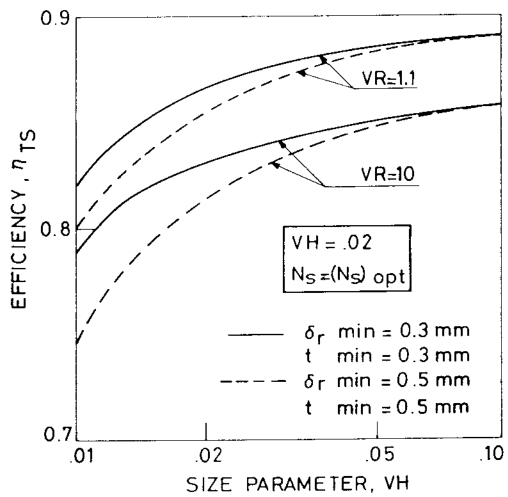


Fig.4: Efficiency penalties as a function of VR and VH, at optimun specific speed.

Fig.5: Total-to-static efficiency for different tip clearance and trailing edge thicknesses, for the extreme values of the expansion ratio.

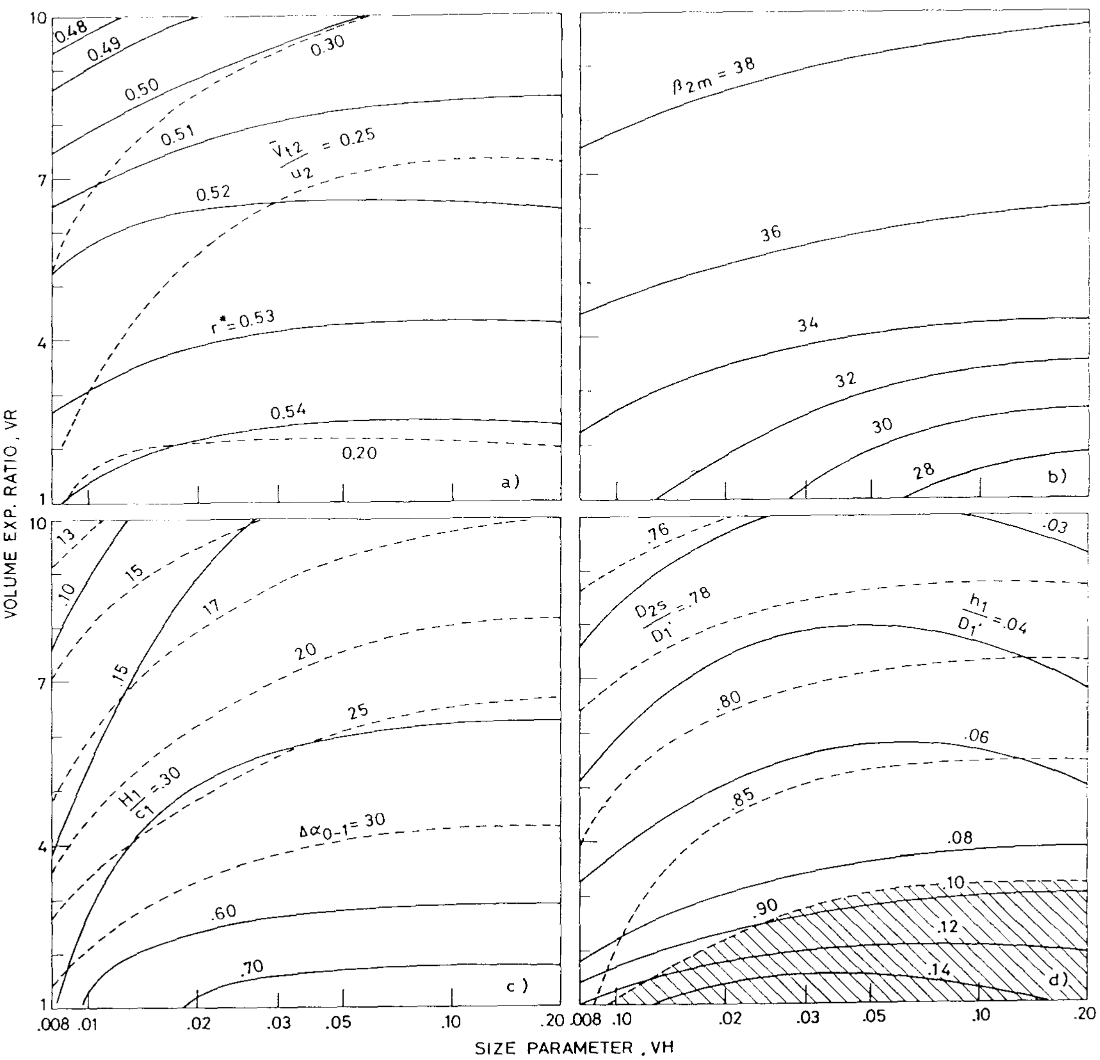


Fig.6: Main geometric and fluid-dynamic features, for optimized turbines, versus VH and VR: a) degree of reaction and swirl coefficient, b) rotor exit blade angle, at mean diameter, c) stator aspect ratio and stator blade deflection, d) rotor exit tip diameter/rotor inlet diameter and stator blade height/rotor inlet diameter.

are related to exit blade height which is large enough, especially at high VR. It must be said that the VH effect is heavily influenced by the assumed values of t, k and δ_r: higher values of these parameters produce larger scale effects, especially at high VR, as shown in Fig.5.

As far as the main fluid dynamic variables are concerned, it can be seen from Fig.6a that the optimum degree of reaction varies from .54 for the incompressible case to .48∿.49 for small turbines with high VR. In the first case higher r* reduces stator losses; in the latter, lower r* provides larger blade height both at rotor inlet and outlet, reducing exit kinetic energy, secondary and leakage losses. The variation of the degree of reaction is not as relevant as in the axial-flow turbines (Macchi and Perdichizzi, 1981) because a large portion of the enthalpy drop is handled by the centripetal work. The optimum head coefficient is not influenced either by compressibility or by size; in fact, it ranges from 2.0 to 2.2 for all design conditions. This implies that relative flow angle at rotor inlet is always close to 90 degrees (radial direction).

An interesting feature shown in Fig.6a is that the adoption of axial absolute velocities at the rotor exit does not produce the maximum total-to-static efficiency, because a considerable amount of swirl (v_{t2}/u_2) is required: in fact, the presence of the swirl implies (i) larger blade exit angle and consequently lower rotor losses, (ii) higher leaving losses, (iii) lower blade exit area, which is important if the design constraints are considered. In fact, the following considerations can be done about the optimum arrangement for the rotor exit angle and the D_{2s}/D_1' ratio (see Fig. 8 b,d): at low VR, high D_{2m}/D_1' occur, because of the small enthalpy drop; in these conditions the maximum D_{2s}/D_1' is obtained in order to provide the maximum exit area and the minimum leaving losses. At high VR, the high enthalpy drops impose small D_{2m}/D_1' ratios: under these conditions, also D_{2s}/D_1' has to be

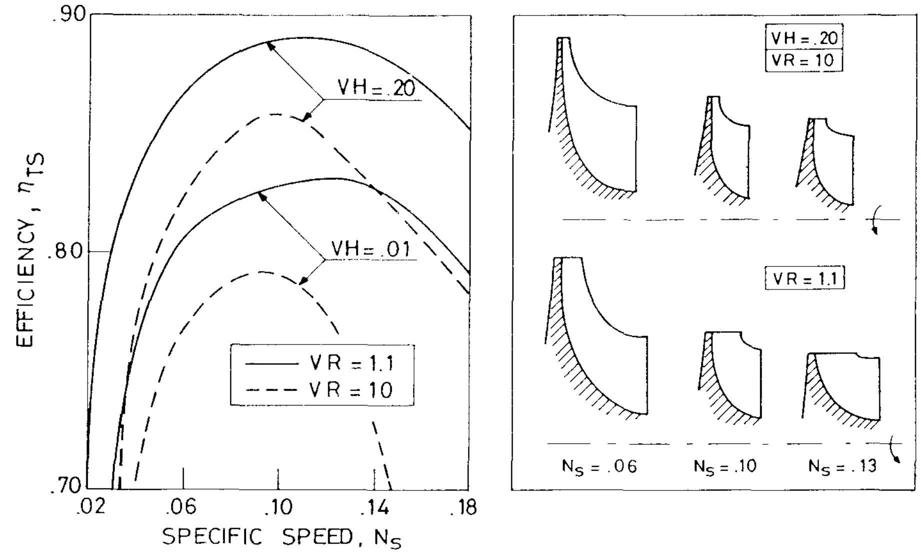


Fig.7: Efficiency prediction for specific speeds different from optimum, and corresponding meridional geometries.

reduced, to keep D_{2h}/D_{2s} within the minimum value compatible with the constraints (0.25); then the volume flow rate must yield higher axial velocities and larger β_2 with higher swirl.

Optimum stator features are presented in Fig. 6 c,d: at high VR, small aspect ratios are predicted, because of the reduced volume flow, together with large secondary losses; lower blade deflections are also being predicted to limit this kind of losses.

6. RESULTS AT NON-OPTIMUM SPECIFIC SPEED

In many applications, the design constraints may impose lower than optimum specific speeds: in fact, the speed of revolution can be previously determined by the driven equipment requirements; in other cases, the speed of revolution derived for optimum N_s can be substantially higher than the maximum imposed by technological limitations (Lozza et al.,1986).

In order to analyze the influence of the specific speed, several turbines were optimized for the extreme expansion ratios and size parameters, at various N_s. The total-to-static efficiency and the meridional geometry of these RIT are presented in Fig.7: small penalties take place for incompressible flows, within a wide range of N_s, while, for high VR, the efficiency rapidly drops, away from the maximum. The loss distribution presented in Fig.8 shows that a decrease of N_s yield an increase in stator losses due to secondary flows, and in disk friction losses, due to larger diameters, while excessive kinetic energy losses take place at N_s higher than the optimum.

Notice that, reducing the specific speed, the larger turbine dimensions cause very large loss coefficients (0.60 for N_s=0.04, dashed line in Fig.8); nevertheless, the rotor loss, in terms of efficiency decrease,is always small ($\Delta\eta \cong 0.04$) and remains constant; this

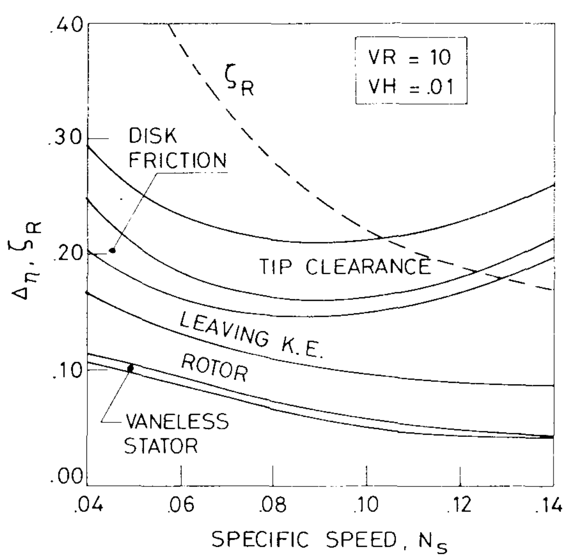


Fig.8: Efficiency penalties and rotor loss coefficients as a function of the specific speed.

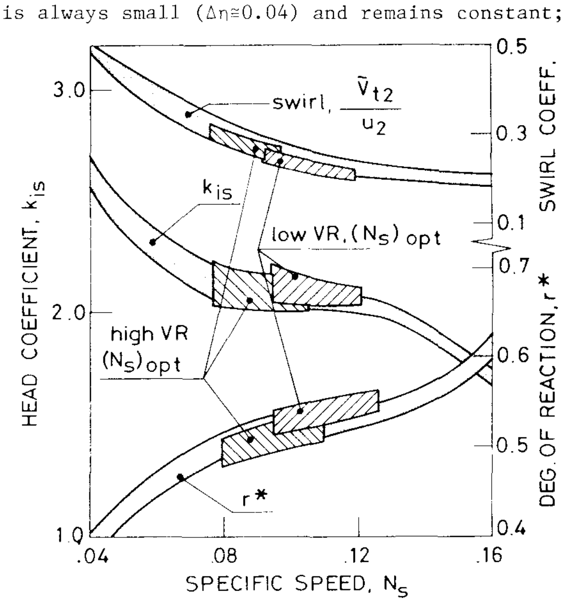


Fig.9: Head coefficient, degree of reaction and swirl coefficient for non-optimum specific speed.

is due to the fact that the rotor loss coefficient is applied to the relative kinetic energy at rotor exit ($w_2^2/2$), which is strongly reduced in respect to the enthalpy drop by centripetal work ($\Delta u^2/2$). This shows that uncertainties in ζ_R evaluation correspond to much smaller unaccuracies in terms of $\Delta\eta_R$.

The optimum values of the main aerodynamic variables (K_{is}, r* and swirl coefficient) for various specific speeds are supplied by Fig.9. Notice that, reducing the specific speed, large head coefficients with small degree of reaction are predicted, to enhance the blade heights and to avoid excessive diameters. Of course, the opposite situation is found at high N_s. The swirl variations (from 0.2 to 0.5) are a consequence of the different r*.

7. CONCLUSIONS

An automatic design procedure has been developed to carry out the optimum design of a radial turbine for specified working conditions. The accuracy of the results, checked through the comparison with experimental efficiencies of various test-cases, was found quite satisfactory.

The optimum turbine efficiency is shown to be influenced both by compressibility and turbine size: the total-to-static efficiency ranges from 0.78 for highly loaded small turbines, to 0.89 for large turbines with small expansion ratios. The optimum specific speed varies from 0.08 to 0.12, depending mainly from the volume expansion ratio; the head coefficient is not influenced by the working conditions (K_{is}=2.0∿2.2); the optimum degree of reaction varies from 0.48 to 0.54 for different VR. An interesting result is that the optimum velocity triangle at the rotor exit does not correspond to zero swirl, but it is obtained for v_{t2}/u_2=0.2∿0.3.

These results allow an optimum turbine design for different operating conditions to be carried out in a simple way: they can be extended to working fluids with different thermodynamic behavior, provided that the required peripheral speed is within the technological limitations imposed by the centrifugal stresses; in fact, in order to obtain more general results, no peripheral speed limit was imposed in the investigation.

ACKNOWLEDGEMENTS

The authors wish to express their gratitude to Prof.Ennio Macchi, for the precious help and the several suggestions. Our thanks also to Mr. R.Biscuola, S.Sensolo and G.Zucchetti for drawing the figures of this paper.

REFERENCES

Ainley,D.C., Mathieson,G.C.R., 1951, "A Method of Performance Estimation for Axial Flow Turbines", British Aeronautical Research Council R&M 294.

Balje', O.E., 1952, Transaction ASME, 741,451.

Benson,R.S., 1970, "A Review of Methods for Assessing Loss Coefficients in Radial Gas Turbines", J.Mech. Sciences, Vol.12.

Bridle.E.A. ,Boulter,R.A., 1967,"A Simple Theory for the [illegible]iction of the Losses in Radial Inflow Turbines", Proc.Inst.Mech.Eng., paper 42, 1967.

Civinkas,K.C., Povinelli,L.A., 1984,"Application of Quasi-3D Inviscid Flow and Boundary Layer Analysis to the Hub Shroud Contouring of a Radial Turbine", AIAA 84-1297, AIAA/SAE/ASME Joint Propulsion Conference.

Craig,H.R., Cox,H.J.A., 1971, "Performance Estimation of Axial Flow Turbines", Proc.Inst.Mech.Eng., Vol. 185,32/71,pp.403-423.

Futral,S.M., Wasserbauer,C.A., 1965,"Off Design Performamce Prediction with Experimental Verification for Radial Inflow Turbines", NASA TN D 2621.

Glassman,A.J., 1976, "Computer Program for Design Analysis of Radial Inflow Turbines", NASA TN D 8164.

Khalil,I.M., Tabakoff,W., Hamed,A., 1976,"Losses in Radial-flow Turbines", J.Eng.Power, pp. 364-372.

Kofskey,M.C., Holeski,D.E., 1965, "Cold Performance Evaluation of a 6.02 Inch Radial Inflow Turbine Designed for a 10 kW Shaft Output Brayton Cycle Space Power generation System", NASA TN D 2987.

Kofskey,M.C., Wasserbauer,C.A., 1966,"Experimental Performance Evaluation of a Radial Inflow Turbine over a Range of Specific Speed", NASA TN D 3742.

Kofskey,M.C., Wasserbauer,C.A., 1968,"Aerodynamic Performance of a 4.95 Inch Modified Radial Turbine Designed for a Single Shaft Brayton Cycle Power System", NASA TM X 1615

Kofskey,M.C., Wasserbauer,C.A., 1969,"Experimental Evaluation of a 3.50 Inch Radial Turbine Designed for a 10 kW Space Power System", NASA TN D 5550.

Lozza,G., Macchi,E., Perdichizzi,A., 1986,"Investigation on the Efficiency Potential of Small Steam Turbines of Various Configuration", Proceedings of the 21st IECEC, paper 869312, pp.1367-1373, San Diego, Ca.

Macchi,E., Perdichizzi,A., 1981, "Efficiency Prediction for Axial Flow Turbines Operating with Nonconventional Fluids", J.Eng.Power Vol.103.

McLallin, K.L., Haas,J.E., 1980, "Experimental Performance and Analysis of 15.04 Tip Diameter Radial Inflow Turbine with Work Factor of 1.126 and Thick Blading", NASA Tech. Paper 1730.

Northern Research, 1983, "R&D for Improved Efficiency Small Steam Turbines", Final Report, Contract No. DE-AC03-79 ET 15426, Feb. 1983.

Rohlik,M.C., 1968, "Analytical Determination of Radial Inflow Turbines Design Geometry for Maximum Efficiency, NASA TN D 4384.

Sasaky,S., Takizawa,K., Mizumachi,N., 1977, "Development of Advanced Radial Gas Turbine for Automobile", Joint Gas Turbine Congress, Tokyo.

Vavra,M.H., 1968, "Flow in Radial Turbines", Short Course on Flow in Turbines, Von Karman Institute, Brussels.

Vavra,M.H., 1969, "Axial Flow Turbines", Lecture Series 15, Von Karman Institute, Brussels.

Wasserbauer,C.A., Kofskey,M.G., 1966, "Cold Performance Evaluation of a 4.59 Inch Radial Turbine Designed for a Brayton Cycle Space Power System",NASA TN D 3260.

Wiesner,F.J., 1967, "A Review of Slip Factors for Centrifugal Impellers", J.Eng.Power, Vol.89,pp.558-566.